Program Verification Using Separation Logic

Dino Distefano
Queen Mary University of London

Lecture 1

Goal of the course

Study Separation Logic having automatic verification in mind

Learn how some notions of mathematical logic can be very helpful in reasoning about real world programs

```c
void t1394Diag_CancelIrp(PDEVICE_OBJECT DeviceObject, PIRP Irp)
{
  KIRQL                Irql, CancelIrql;
  BUS_RESET_IRP        *BusResetIrp, *temp;
  PDEVICE_EXTENSION    deviceExtension;

  deviceExtension = DeviceObject->DeviceExtension;

  KeAcquireSpinLock(&deviceExtension->ResetSpinLock, &Irql);

  temp = (PBUS_RESET_IRP)deviceExtension;
  BusResetIrp = (PBUS_RESET_IRP)deviceExtension->Flink2;

  while (BusResetIrp) {

    if (BusResetIrp->Irp == Irp) {
      temp->Flink2 = BusResetIrp->Flink2;
      free(BusResetIrp);
      break;
    }
    else if (BusResetIrp->Flink2 == (PBUS_RESET_IRP)deviceExtension) {
      break;
    }
    else {
      temp = BusResetIrp;
      BusResetIrp = (PBUS_RESET_IRP)BusResetIrp->Flink2;
    }
  }

  KeReleaseSpinLock(&deviceExtension->ResetSpinLock, Irql);

  IoReleaseCancelSpinLock(Irp->CancelIrql);
  Irp->IoStatus.Status = STATUS_CANCELLED;
  IoCompleteRequest(Irp, IO_NO_INCREMENT);
} // t1394Diag_CancelIrp
```

A piece of a windows device driver.

Is this correct?

Or at least: does it have basic properties like it won't crash or leak memory?

Today's plan

- Motivation for Separation Logic
- Assertion language
- Mathematical model
- Data structures

Motivations...

Simple Imperative Language

- Safe commands:
  - S::= skip | x:=E | x:=new(E1,...,En)
- Heap accessing commands:
  - A(E) ::= dispose(E) | x:=[E] | [E]:=F

where E is an expression e.g., x, y, nil, etc.

- Command:
  - C::= S | A | C1;C2 | if B { C1 } else {C2} | while B do { C }

where B boolean guard E=E, E!=E, etc.

Example Program: List Reversal

```
p:=nil;
while (c !=nil) do {
  t:=p;
  p:=c;
  c:=[c];
  [p]:=t;
}
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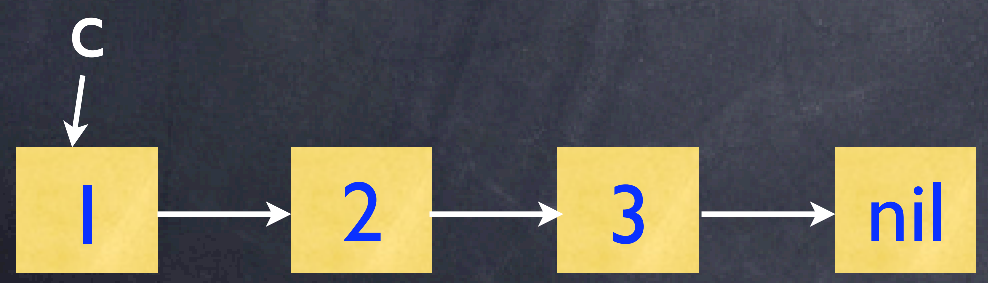

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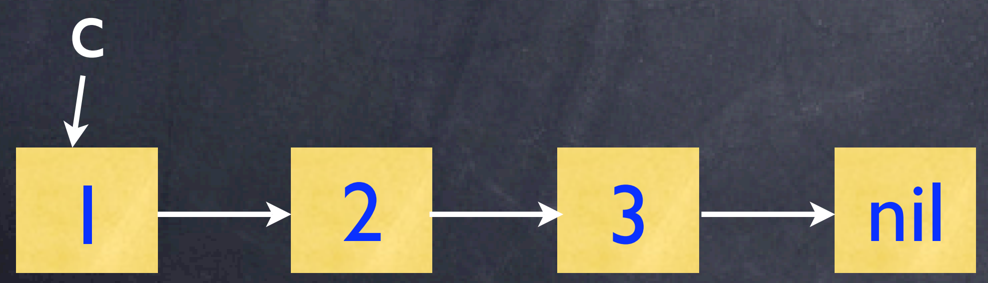

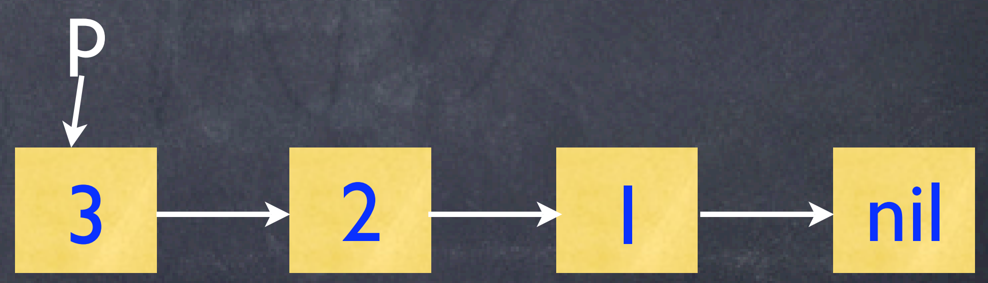

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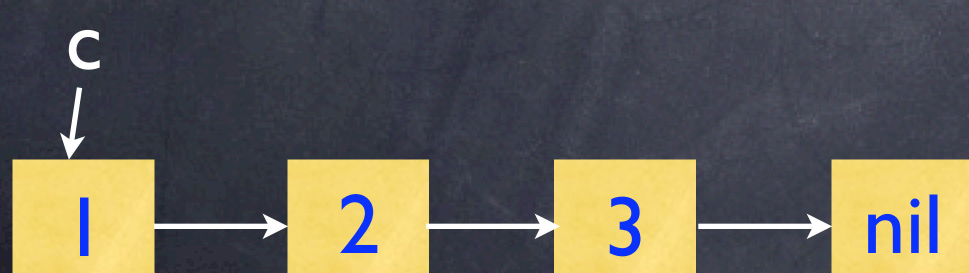
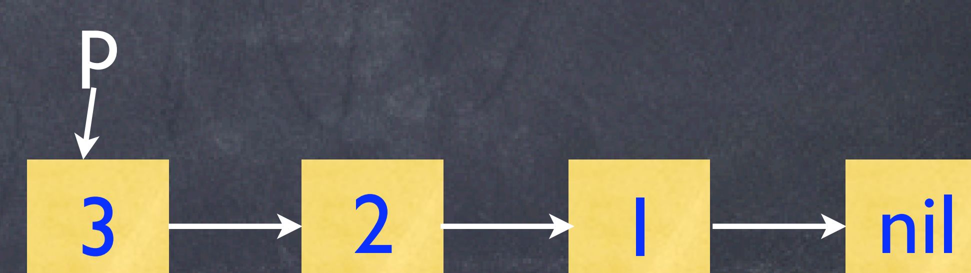
Some properties we would like to prove:

Does the program preserve acyclicity/cyclicity?

Does it core-dump?

Does it create garbage?

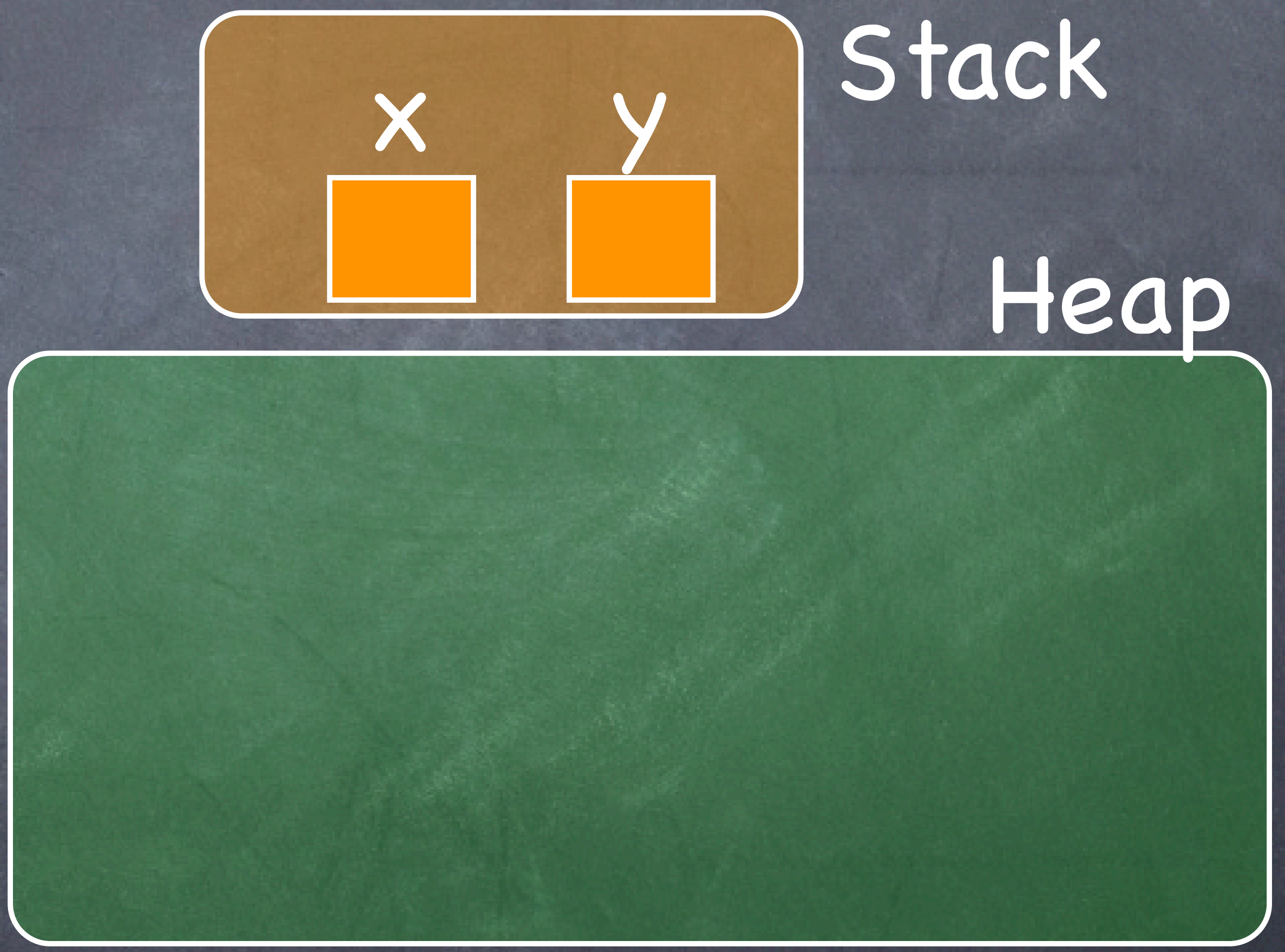




Example Program

We are interested in pointer manipulating programs

```
x = new(3,3);
y = new(4,4);
[x+1] = y;
[y+1] = x;
y = x+1;
dispose x;
y = [y];
```



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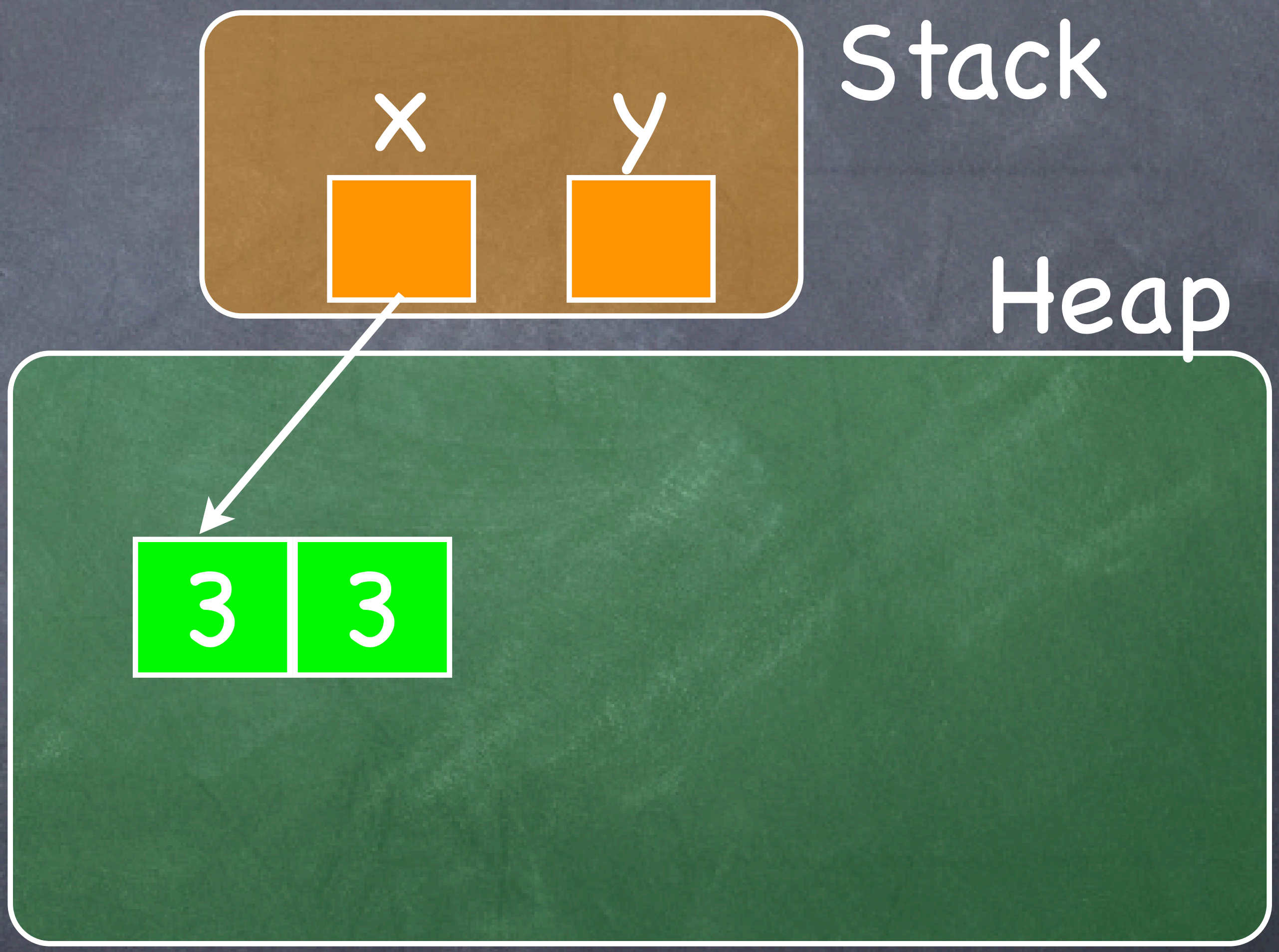

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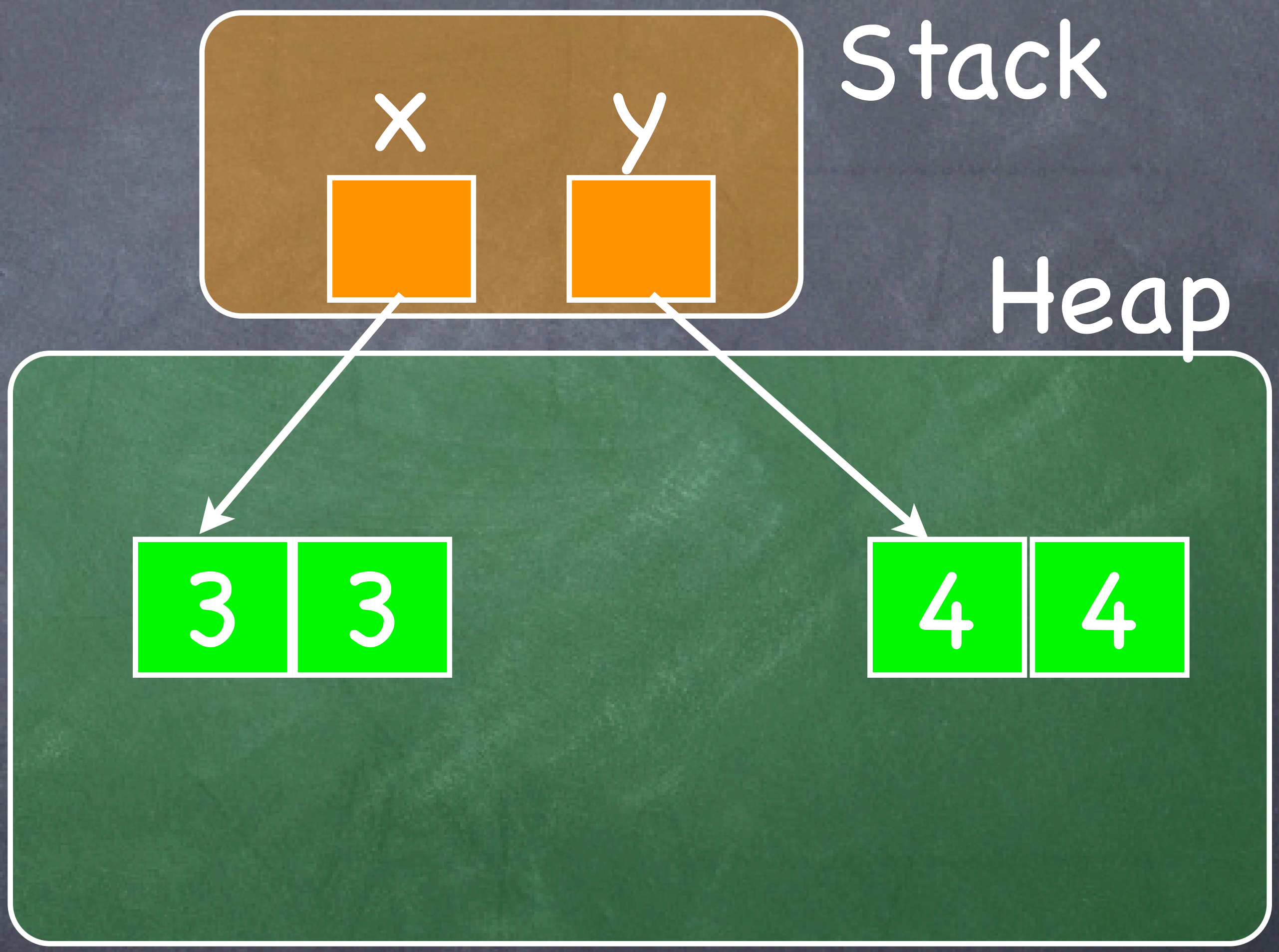

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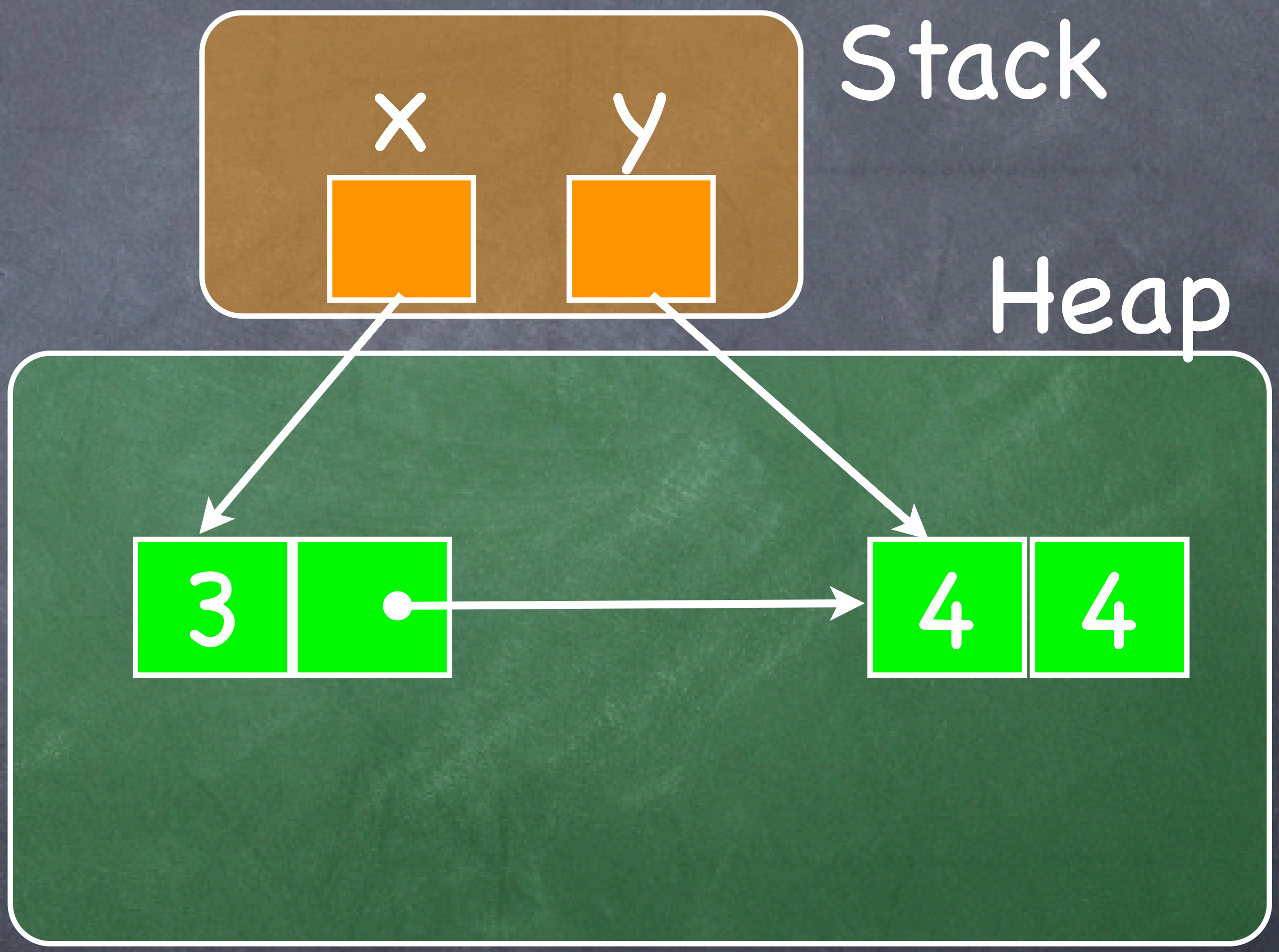

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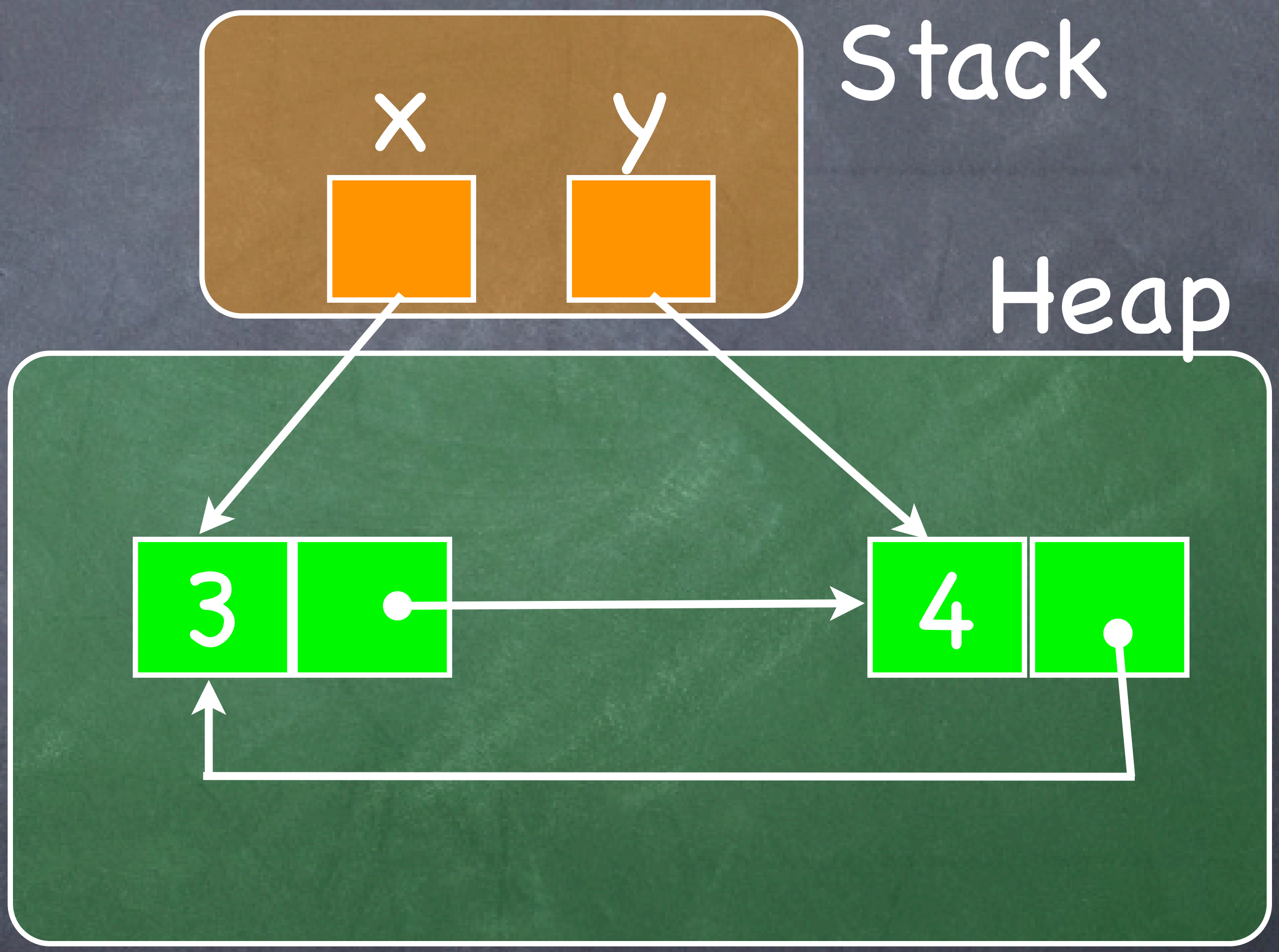

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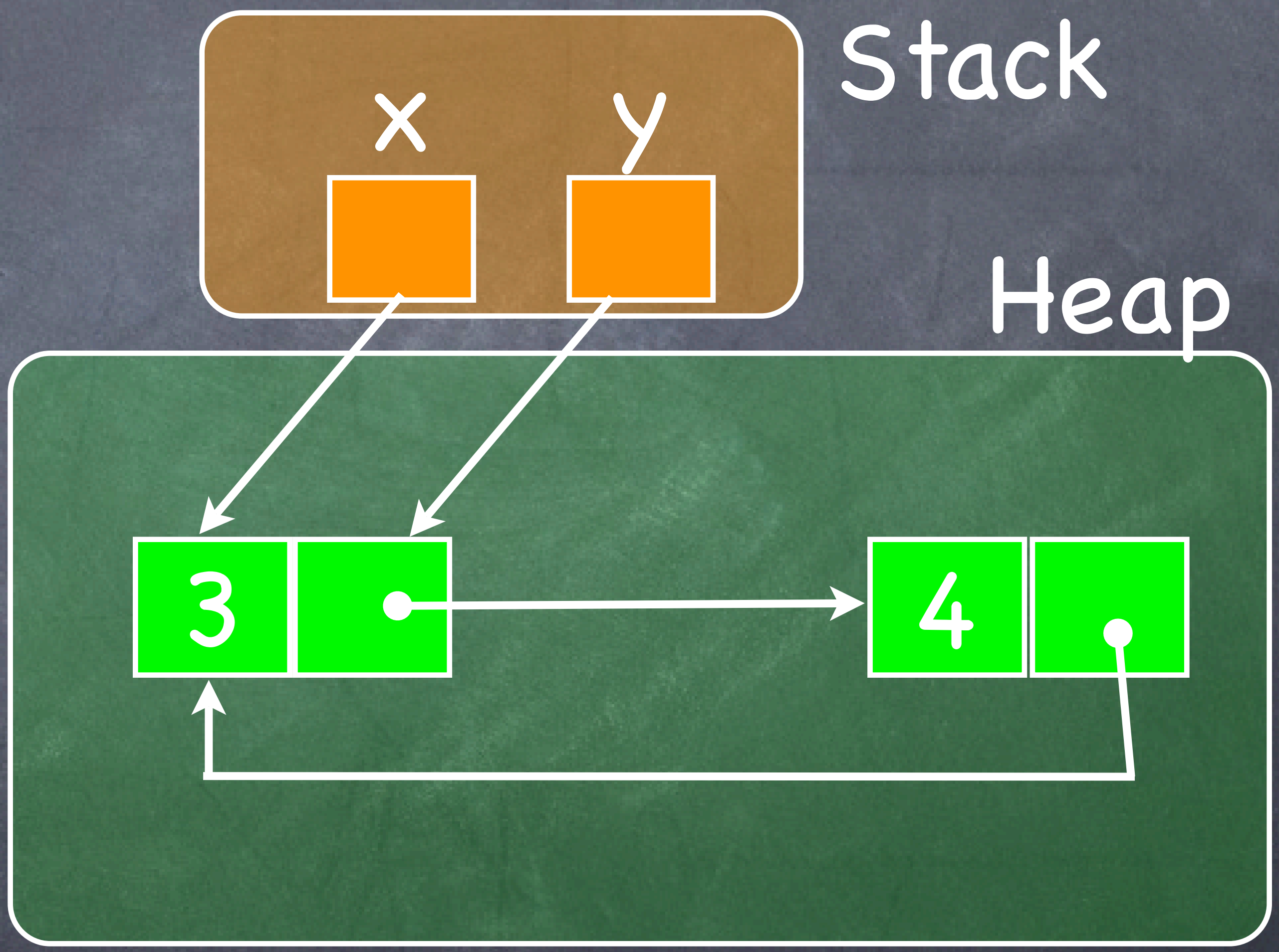

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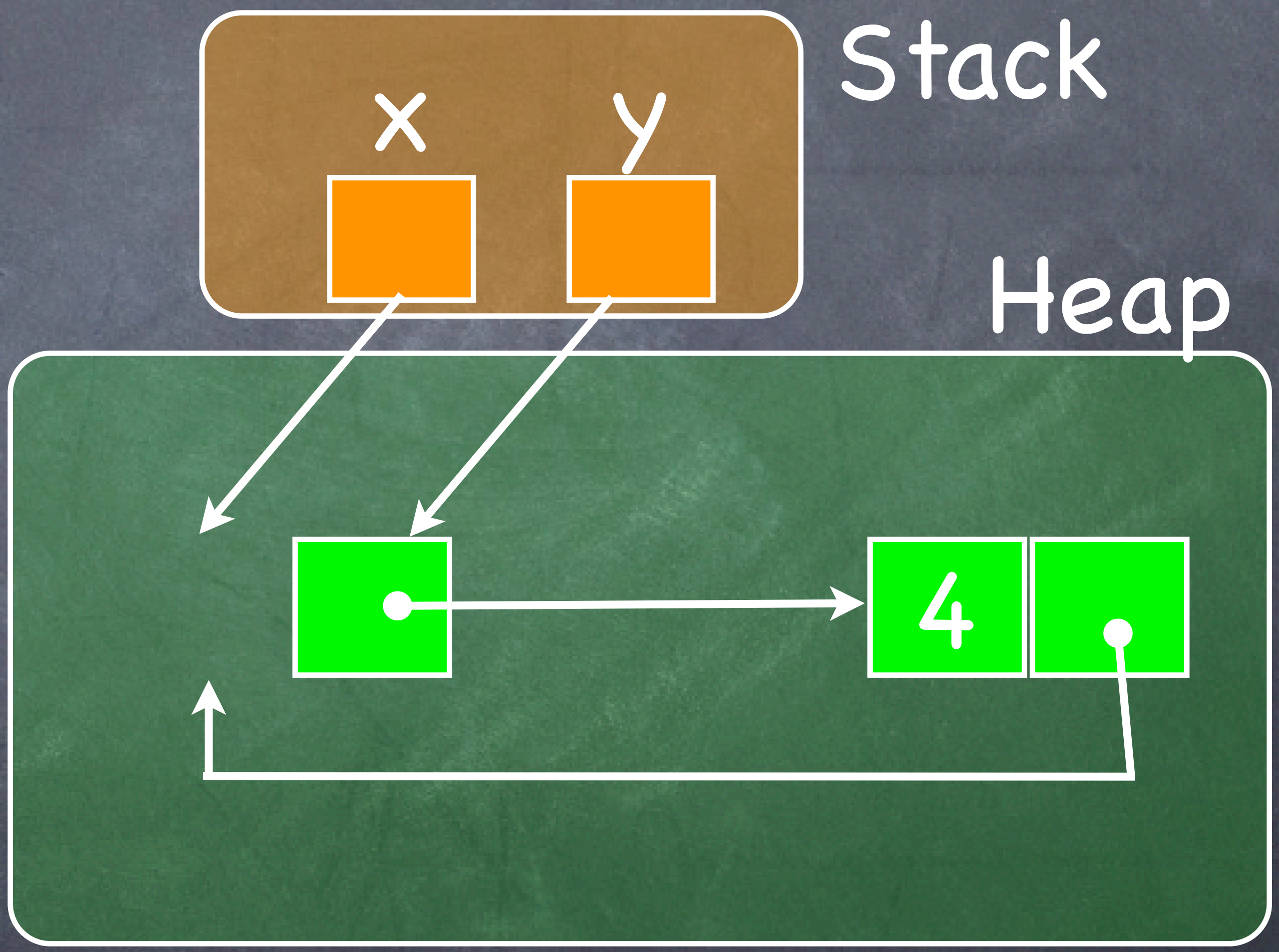

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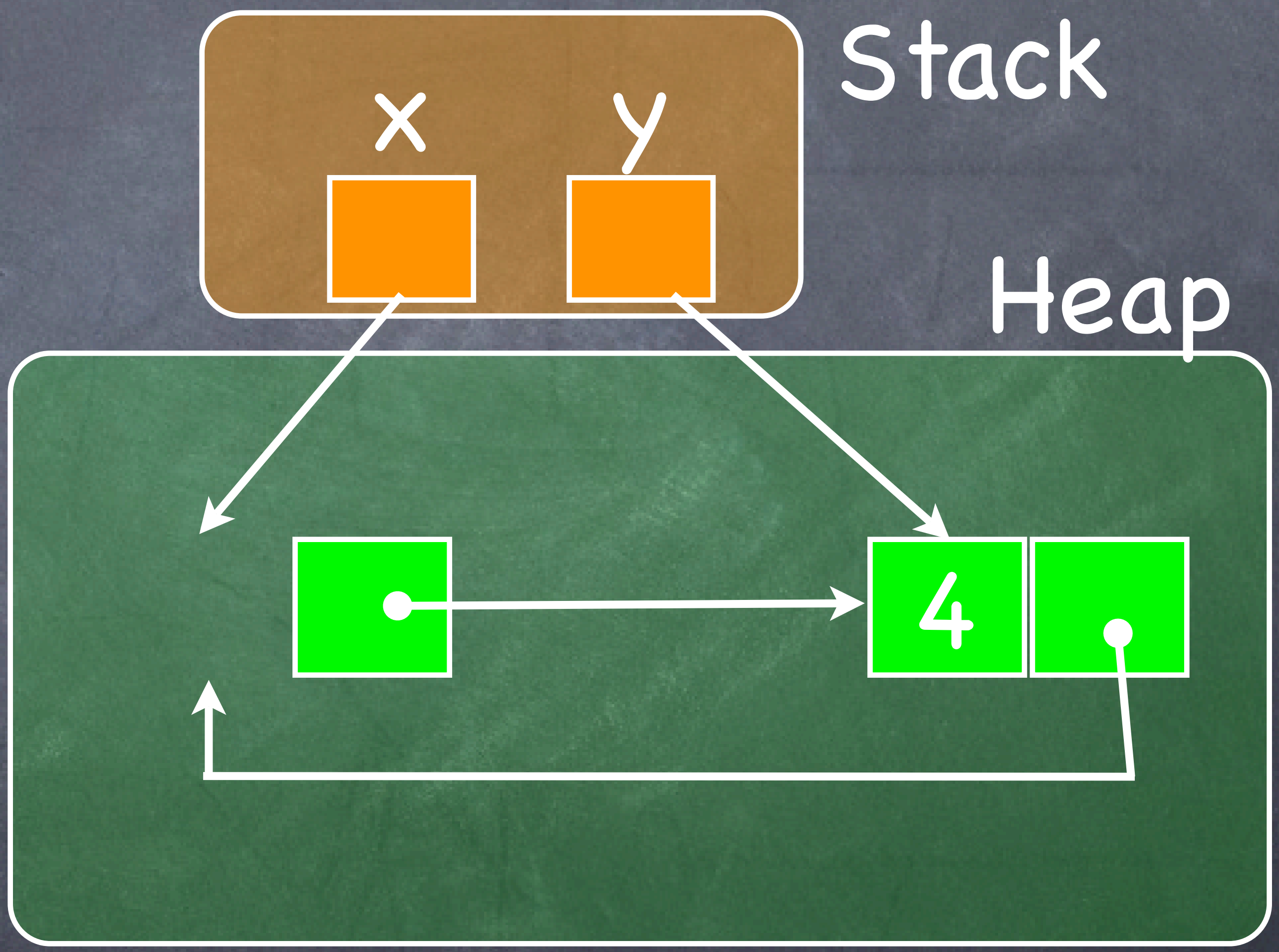

Why Separation Logic?

Consider this code:

```
[y] = 4;
[z] = 5;
Guarantee([y] != [z])
```

We need to know that things are different. How?

Why Separation Logic?

Consider this code:

```
Assume(y != z)
  [y] = 4;
  [z] = 5;
Guarantee([y] != [z])
```

Add assertion?

We need to know that things are different. How?

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Consider this code:

Assume(y != z)

Add assertion?

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  [y] = 4;
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We need to know that things are different. How?

We need to know that things stay the same. How?

Why Separation Logic?

Consider this code:

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Assume([x] = 3)
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Guarantee([x] = 3)
```

Add assertion?

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We need to know that things stay the same. How?

Why Separation Logic?

Consider this code:

Assume([x] = 3 && x!=y && x!=z) Add assertion?
Assume(y != z) Add assertion?

```
  [y] = 4;
  [z] = 5;
```

Guarantee([y] != [z])
Guarantee([x] = 3)

We need to know that things are different. How?
We need to know that things stay the same. How?

Framing

We want a general concept of things not being affected.

$$\frac{\{P\}\ C\ \{Q\}}{\{R\ \&\&\ P\}\ C\ \{Q\ \&\&\ R\}}$$

What are the conditions on C and R?

Hard to define if reasoning about a heap and aliasing

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What are the conditions on C and R?

Hard to define if reasoning about a heap and aliasing

This is where separation logic comes in

$$\frac{\{P\}\ C\ \{Q\}}{\{R * P\}\ C\ \{Q * R\}}$$

Introduces new connective * used to separate state.

The Logic

Storage Model

$$\mathsf{Vars} \stackrel{def}{=} \{x, y, z, \ldots\}$$

$$\mathsf{Locs} \stackrel{def}{=} \{1, 2, 3, 4, \ldots\} \qquad \mathsf{Vals} \supseteq \mathsf{Locs}$$

$$\mathsf{Heaps} \stackrel{def}{=} \mathsf{Locs} \rightharpoonup_{\mathsf{fin}} \mathsf{Vals}$$

$$\mathsf{Stacks} \stackrel{def}{=} \mathsf{Vars} \to \mathsf{Vals}$$

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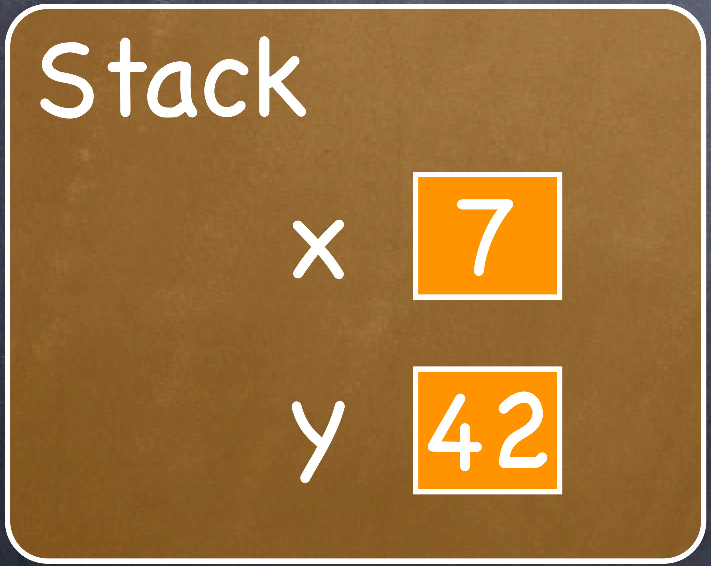

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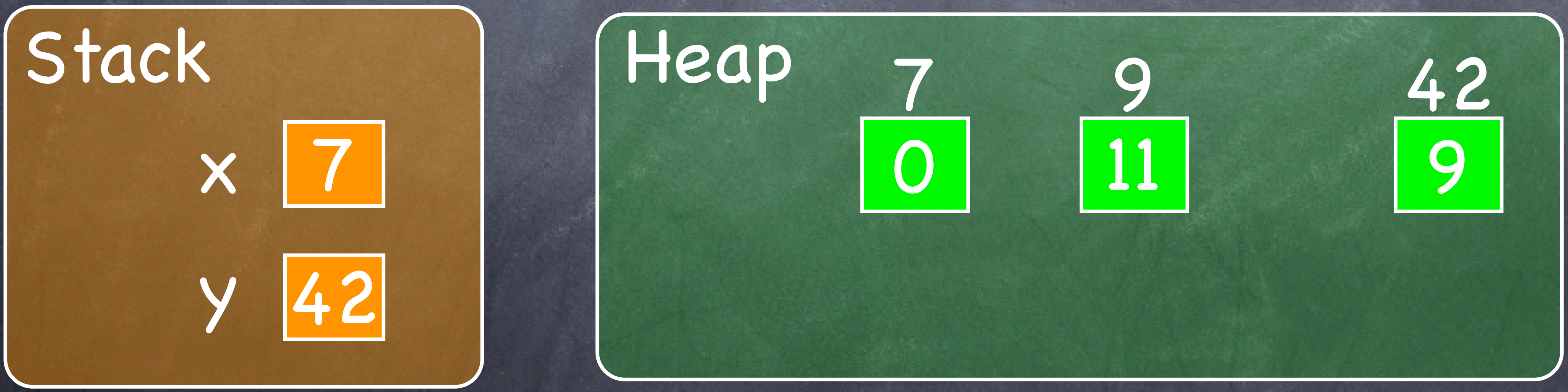

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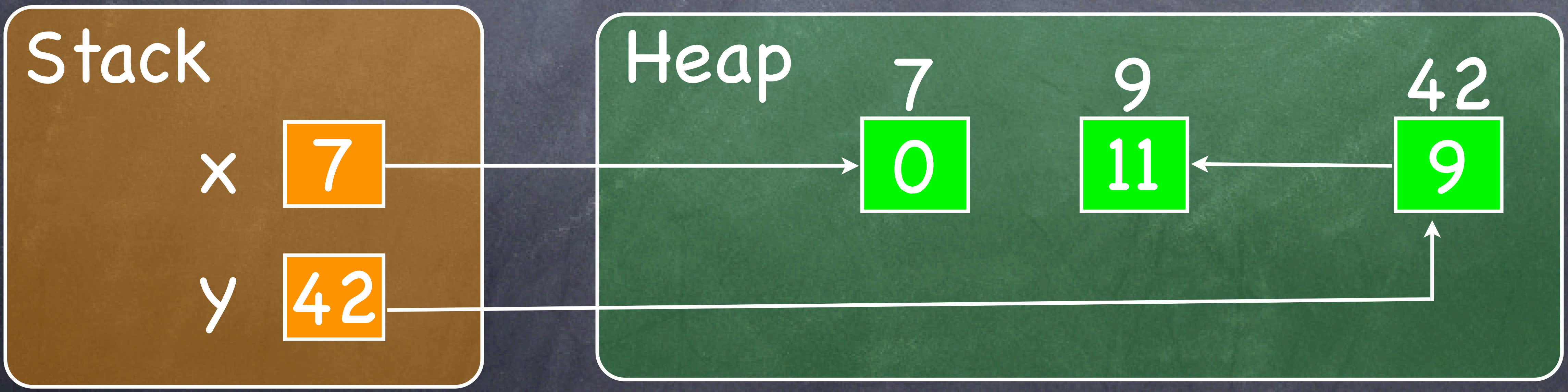

Mathematical Structure of Heap

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$$h_1 \# h_2 \quad \stackrel{def}{\Longleftrightarrow} \quad \mathsf{dom}(h_1) \cap \mathsf{dom}(h_2) = \emptyset$$

$$h_1 * h_2 \quad \stackrel{def}{=} \quad \begin{cases} h_1 \cup h_2 & \text{if } h_1 \# h_2 \\ \text{undefined} & \text{otherwise} \end{cases}$$

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1) * has a unit

2) * is associative and commutative

3) (Heap,*,{}) is a partial commutative monoid

Assertions

$$
\begin{array}{rcll}
E, F & ::= & x \mid n \mid E{+}F \mid -E \mid \ldots & \text{Heap-independent Exprs} \\
P, Q & ::= & E = F \mid E \geq F \mid E \mapsto F & \text{Atomic Predicates} \\
 & \mid & \mathsf{emp} \mid P * Q & \text{Separating Connectives} \\
 & \mid & \mathsf{true} \mid P \wedge Q \mid \neg P \mid \forall x.\, P & \text{Classical Logic}
\end{array}
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Informal Meaning

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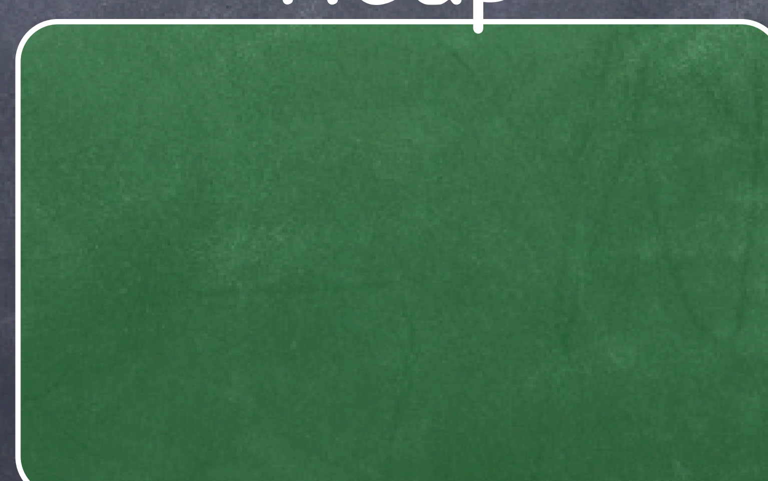

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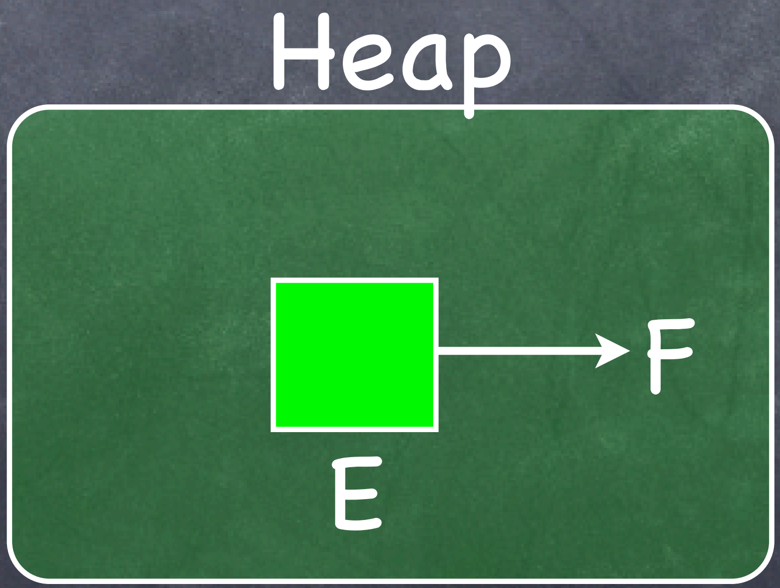

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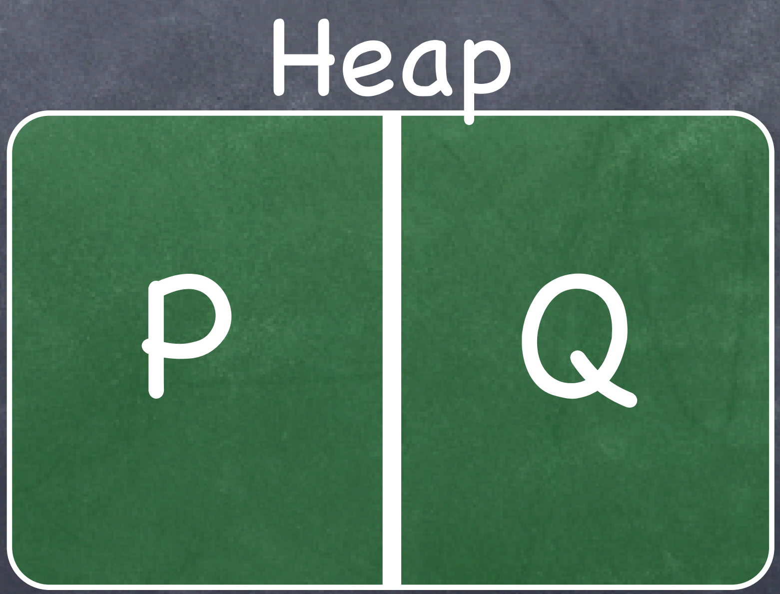
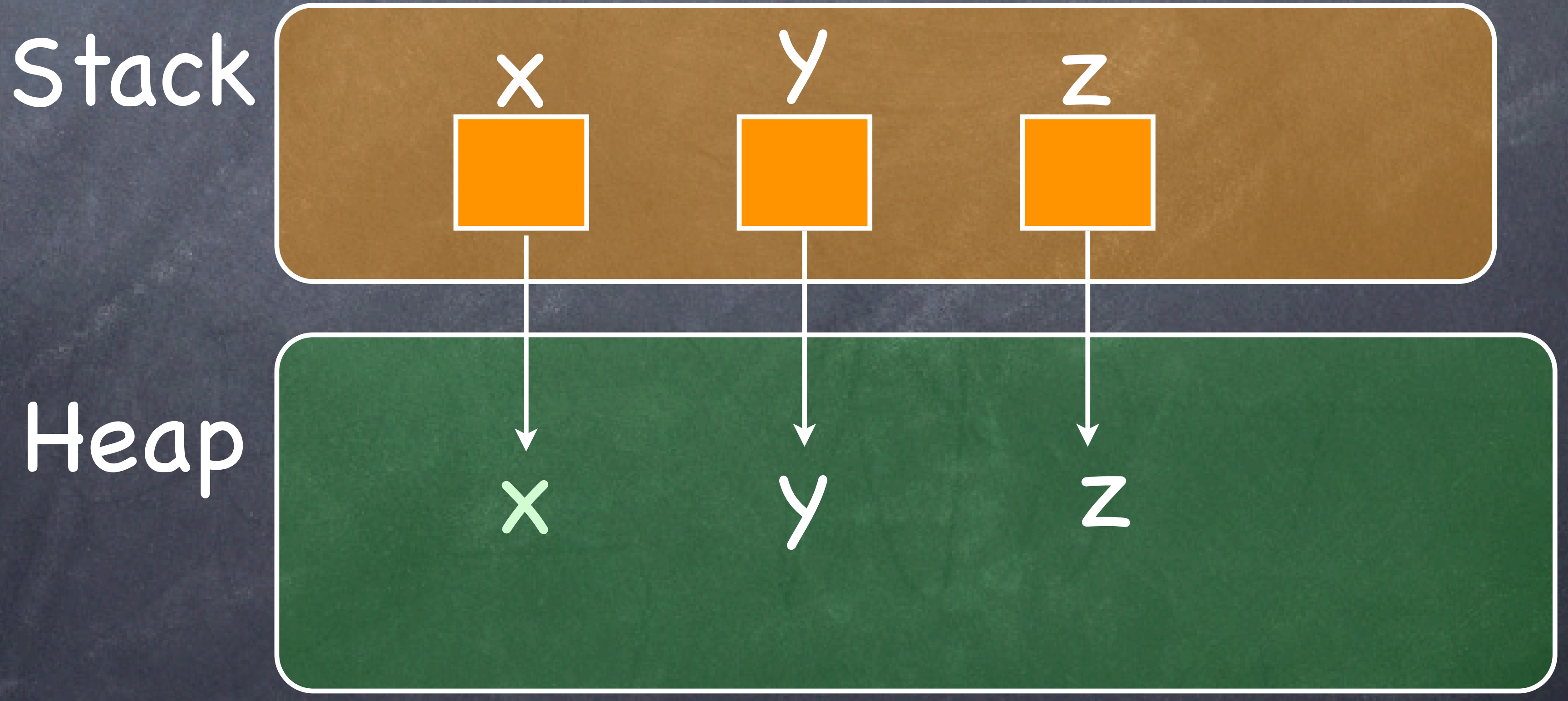


Examples

Formula: emp



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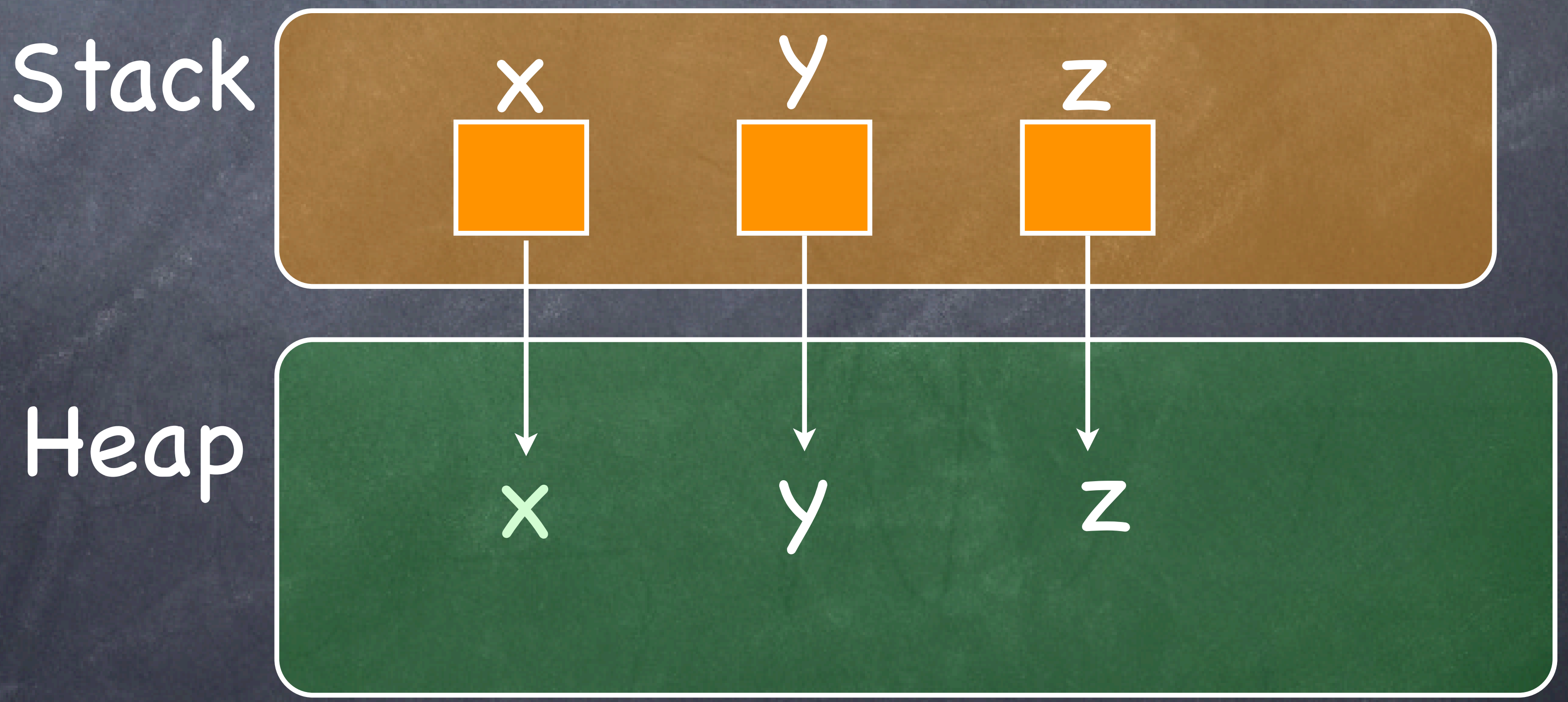
Formula: emp*x|->y



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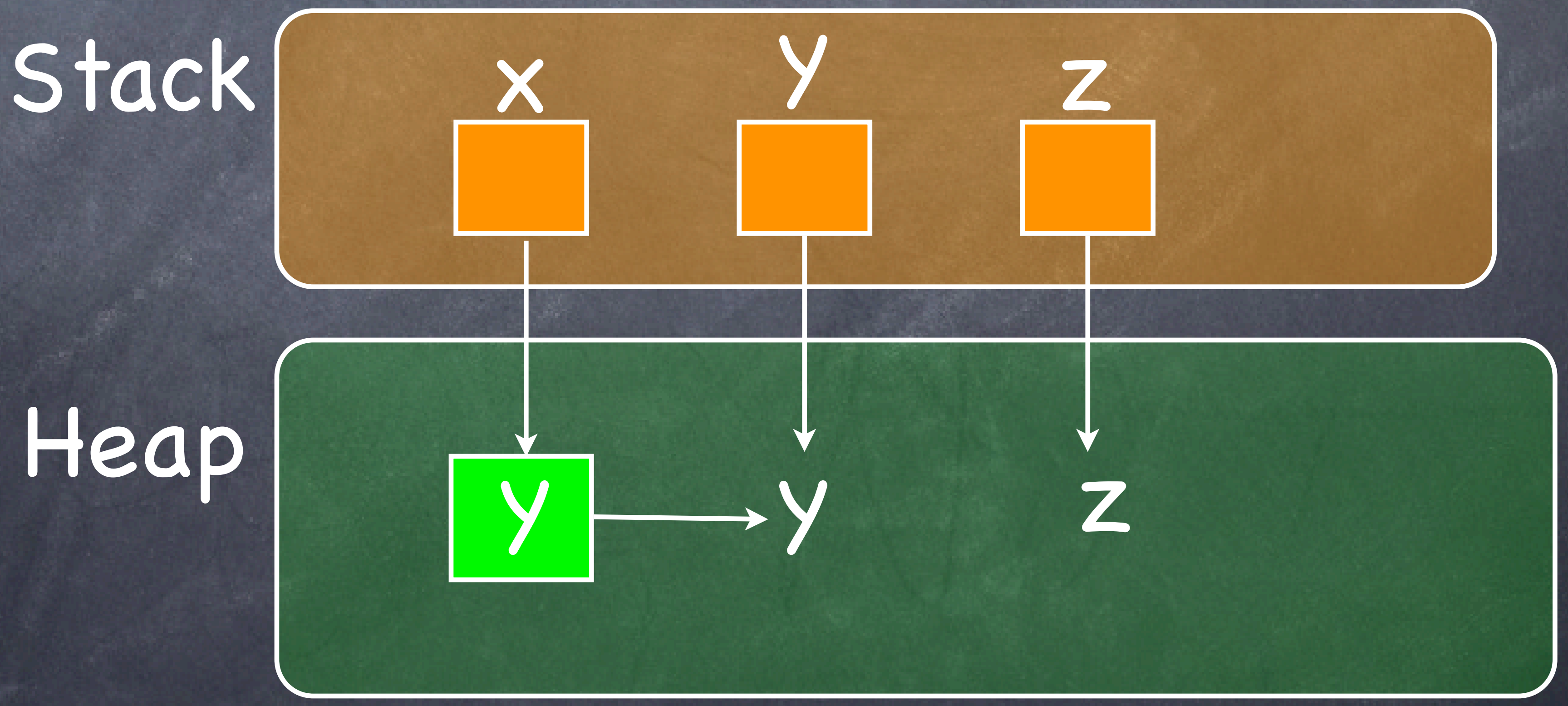

Examples

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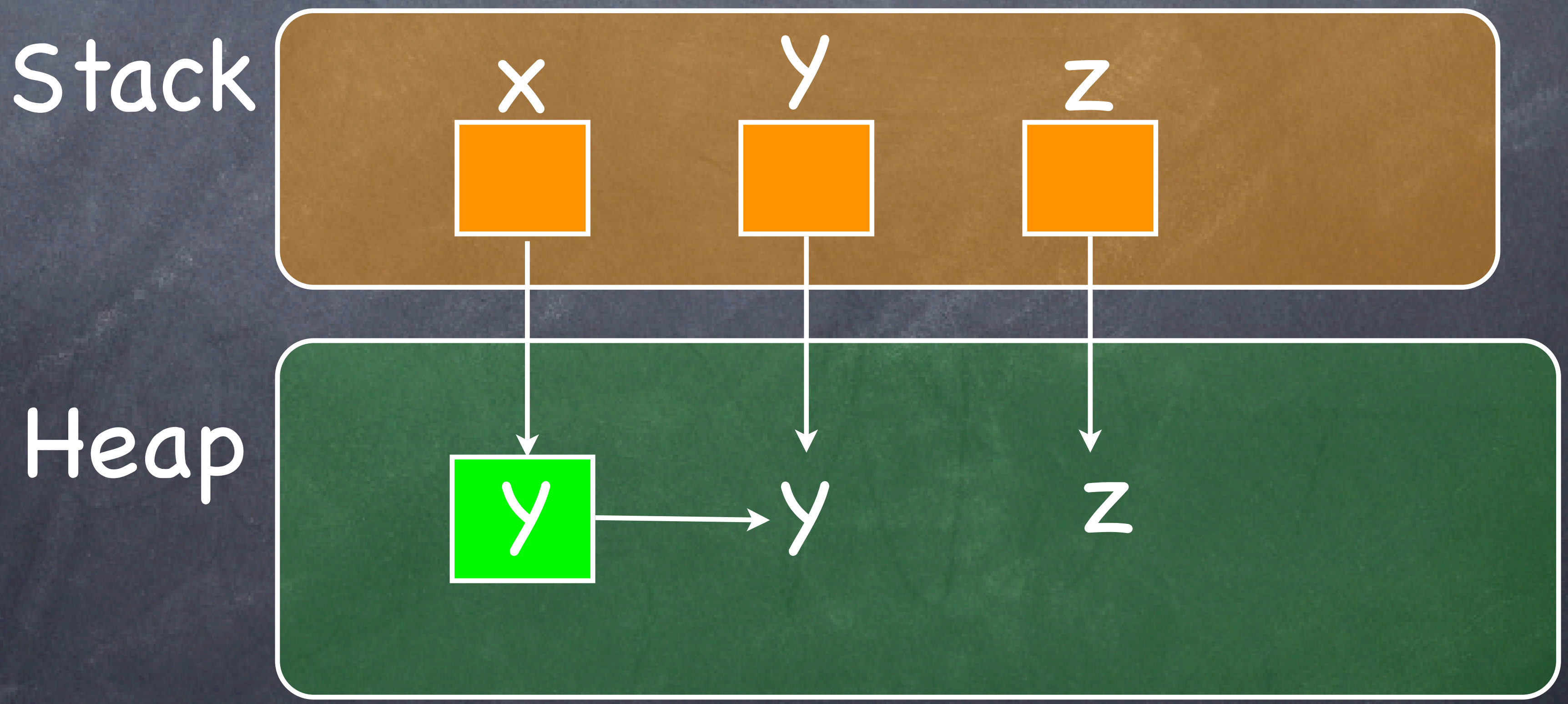
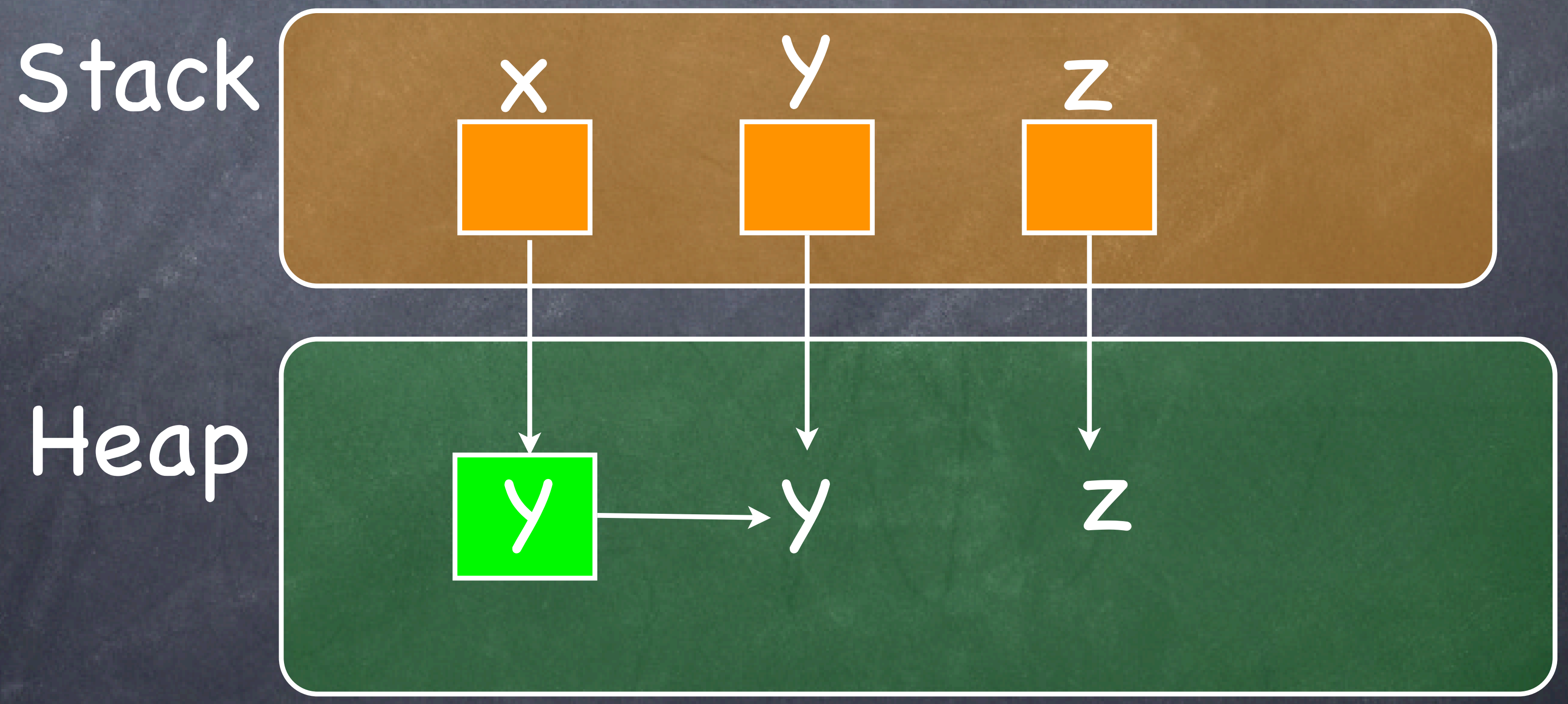


Examples

Formula: x|->y * y|->z



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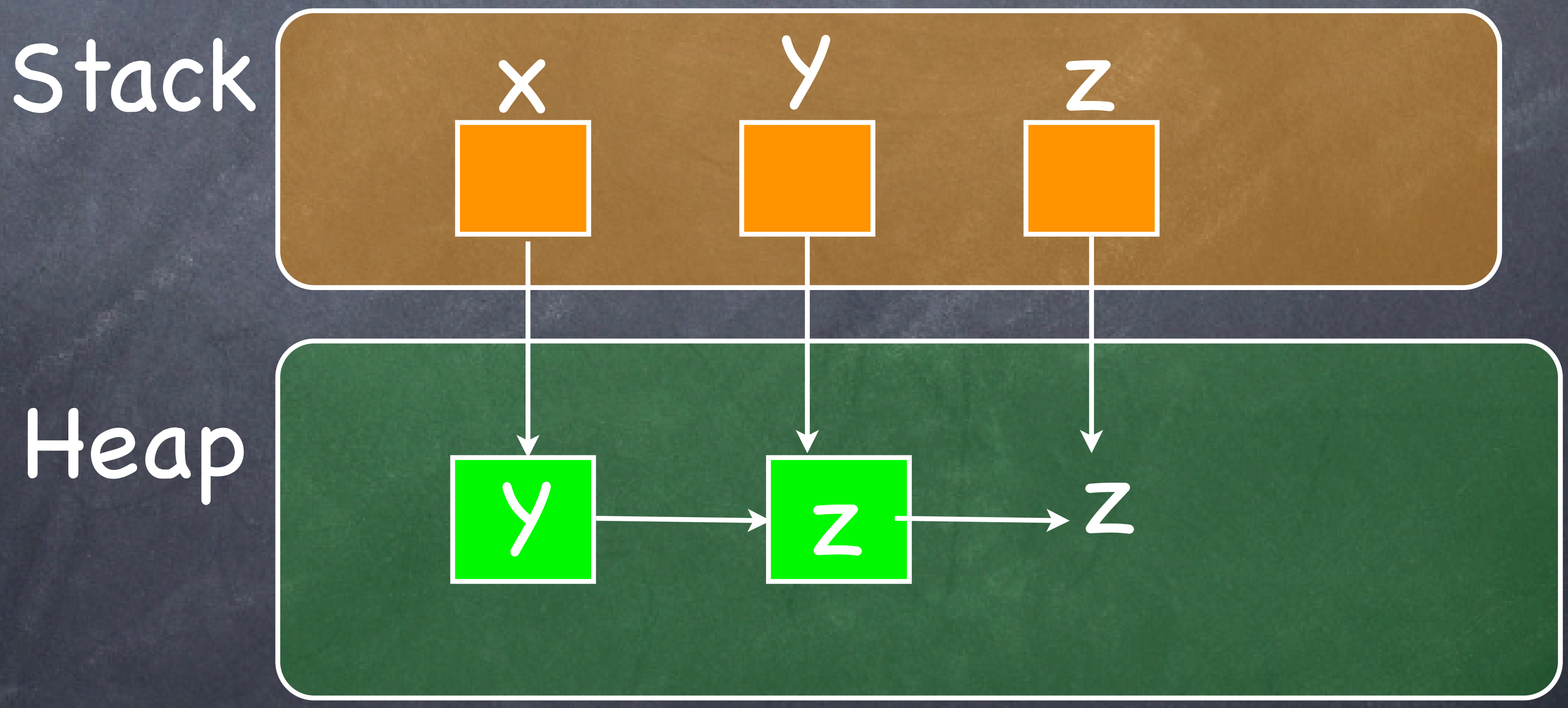
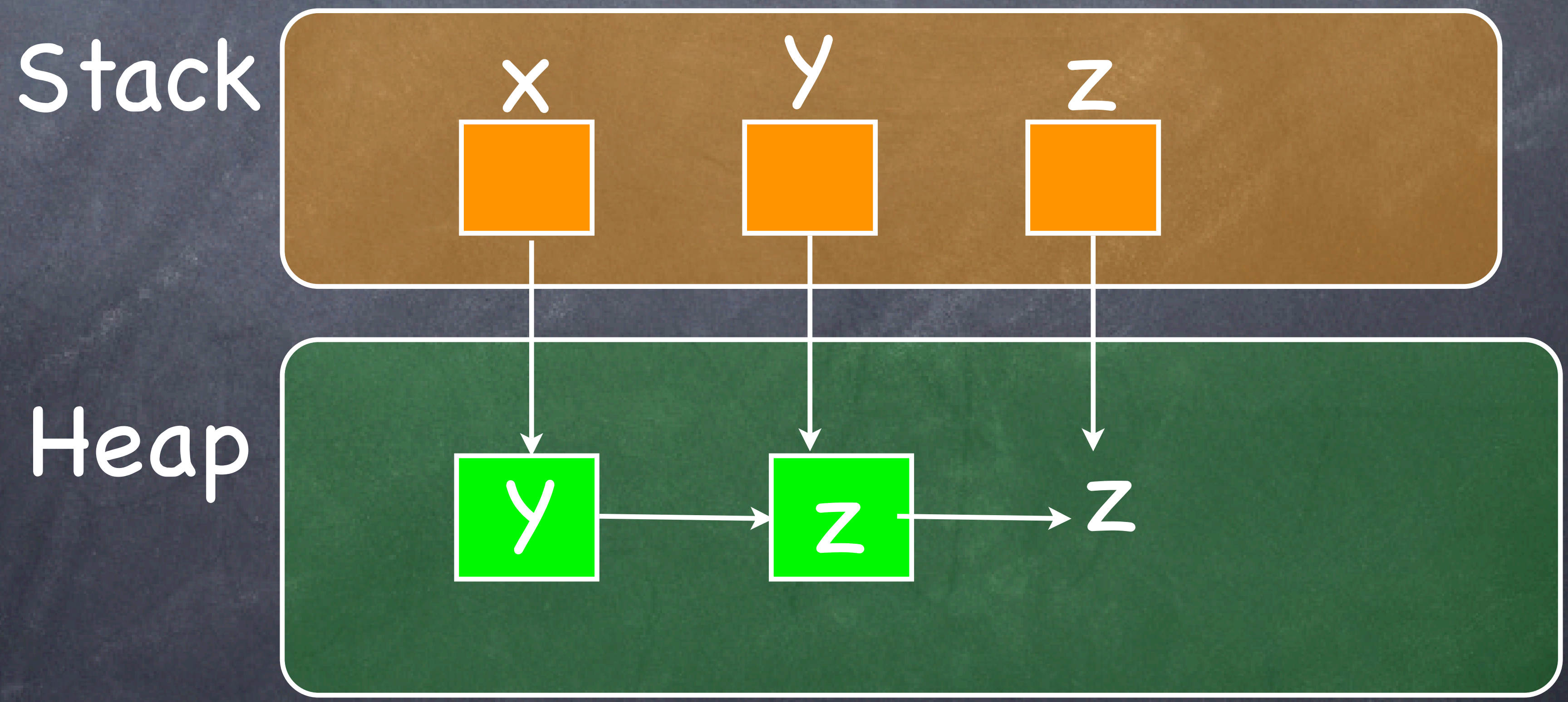


Examples

Formula: x|->y * y|->z* z|->x



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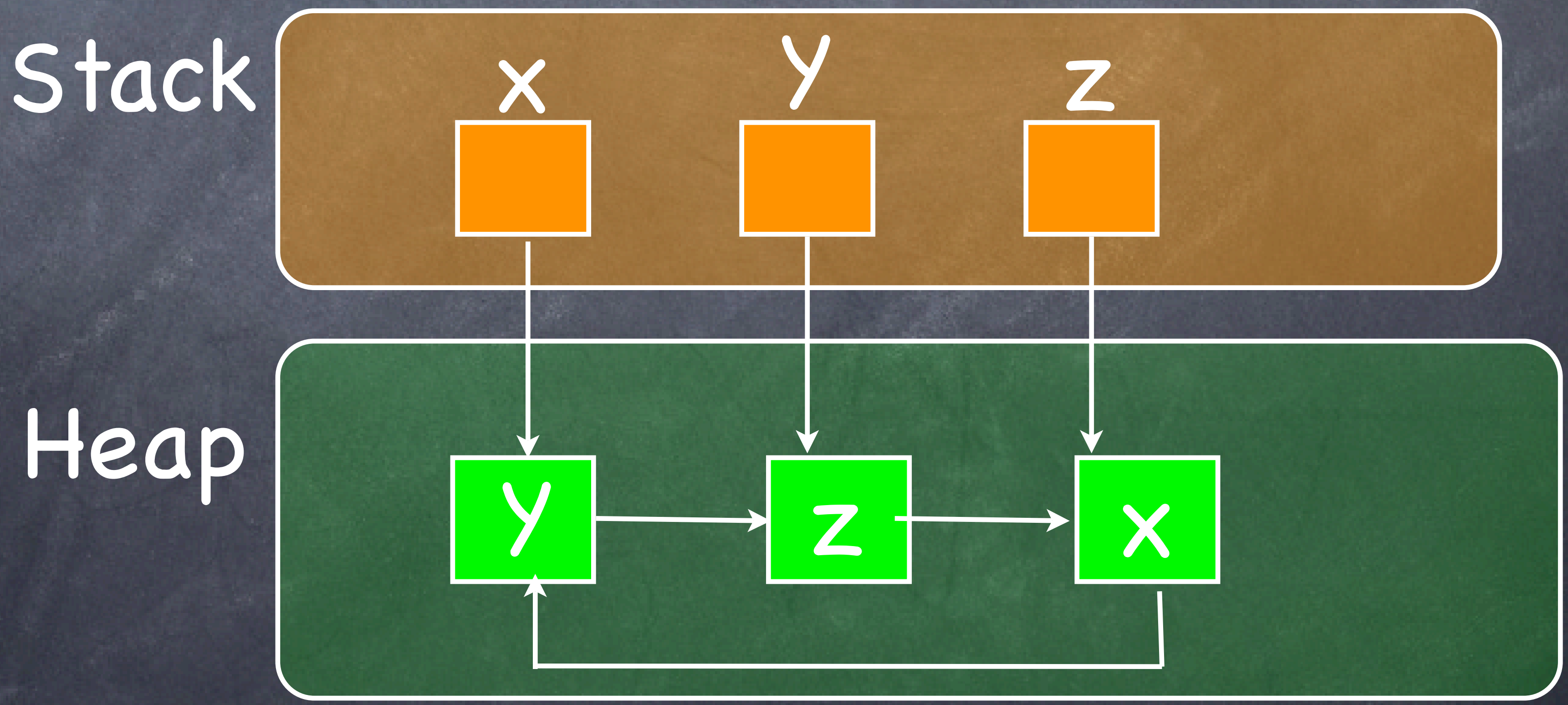

Semantics of Assertions

- Expressions mean maps from stacks to integers.

$$[\![E]\!] \quad : \quad \mathsf{Stacks} \longrightarrow \mathsf{Vals}$$

- Semantics of assertions given by satisfaction relation between states and assertions.

$$(s, h) \quad \models \quad P$$

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Stack

Heap

Semantics of Assertions

$(s,h) \models E \geq F$	iff	$[\![E]\!]s, [\![F]\!]s \in \mathsf{Integers}$ and $[\![E]\!]s \geq [\![F]\!]s$
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$(s,h) \models P * Q$	iff	$\exists h_0 h_1.\ h_0 * h_1 = h,\ (s,h_0) \models P$ and $(s,h_1) \models Q$
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Abbreviations

The address E is active:

$$E \mapsto - \triangleq \exists x'.E \mapsto x'$$

where x′ not free in E

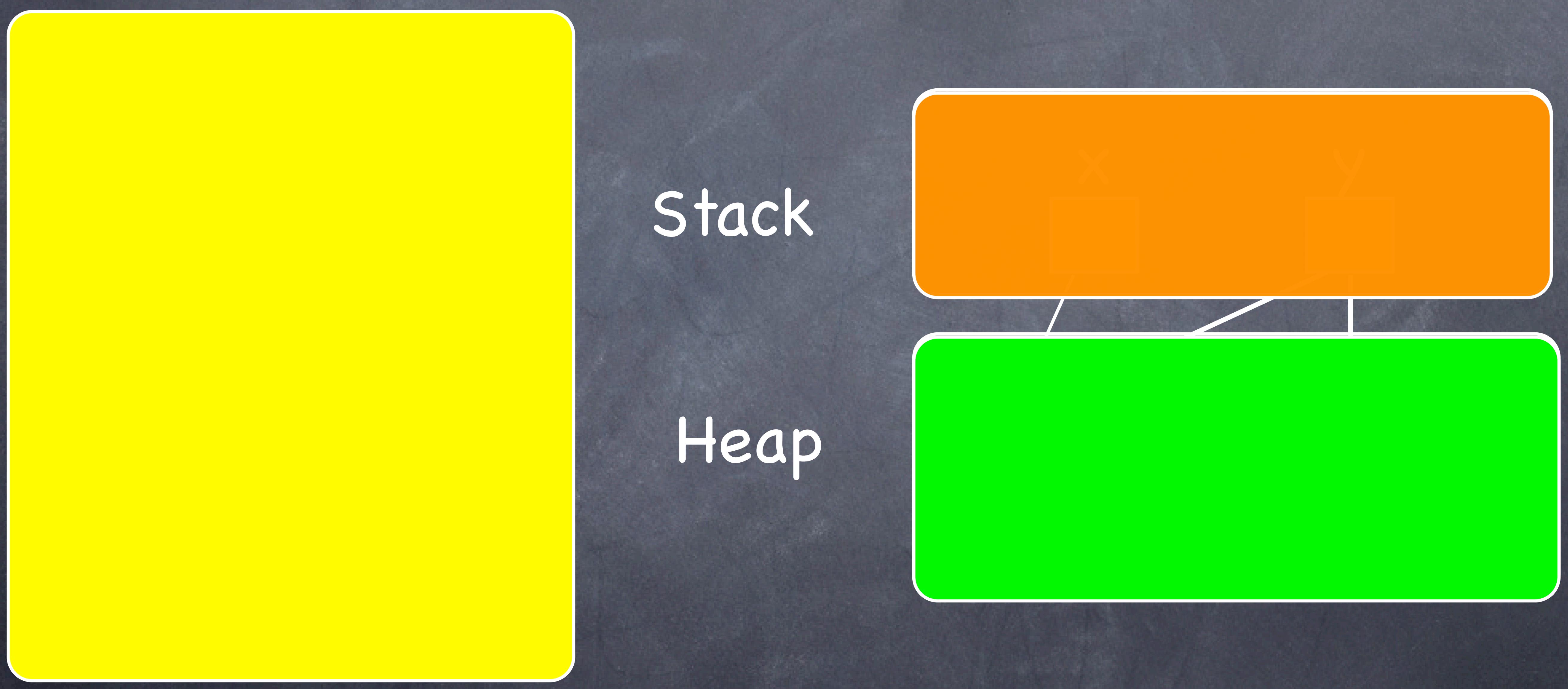
E points to F somewhere in the heap:

$$E \hookrightarrow F \triangleq E \mapsto F * \mathsf{true}$$

E points to a record of several fields:

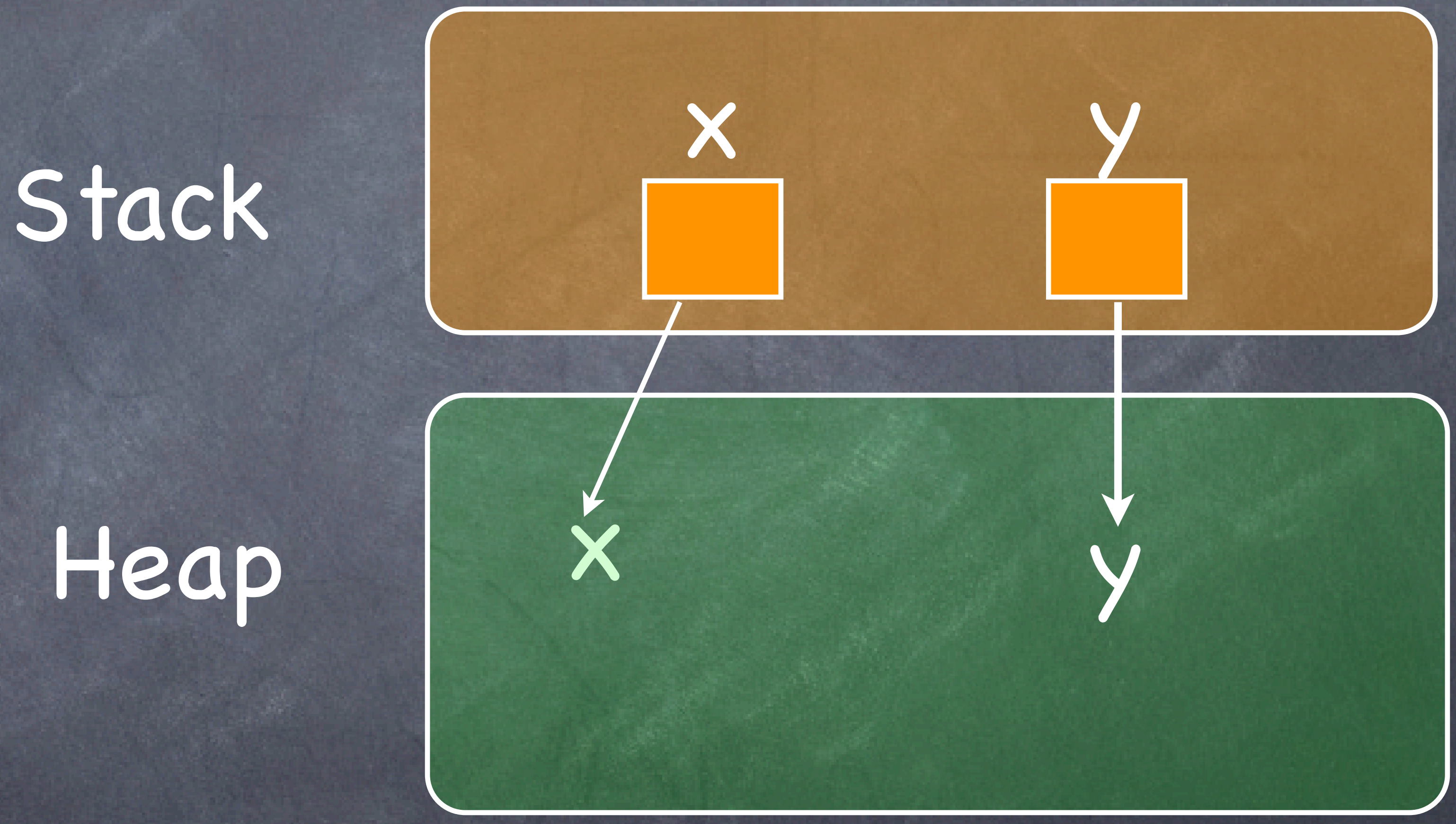
$$E \mapsto E_1, \ldots, E_n \triangleq E \mapsto E_1 * \cdots * E + n - 1 \mapsto E_n$$

Example



Example

$x \mapsto 3, y$



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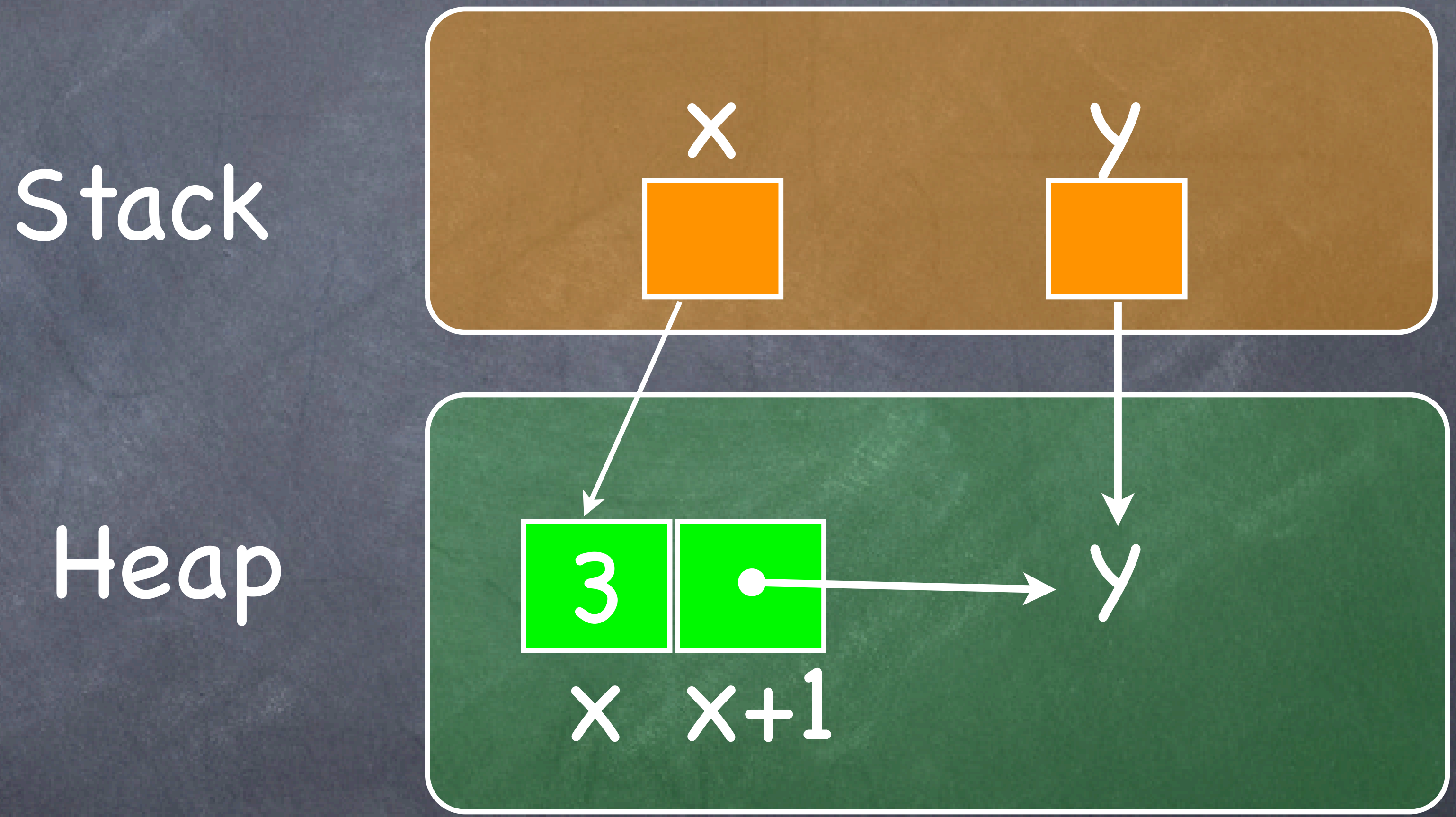

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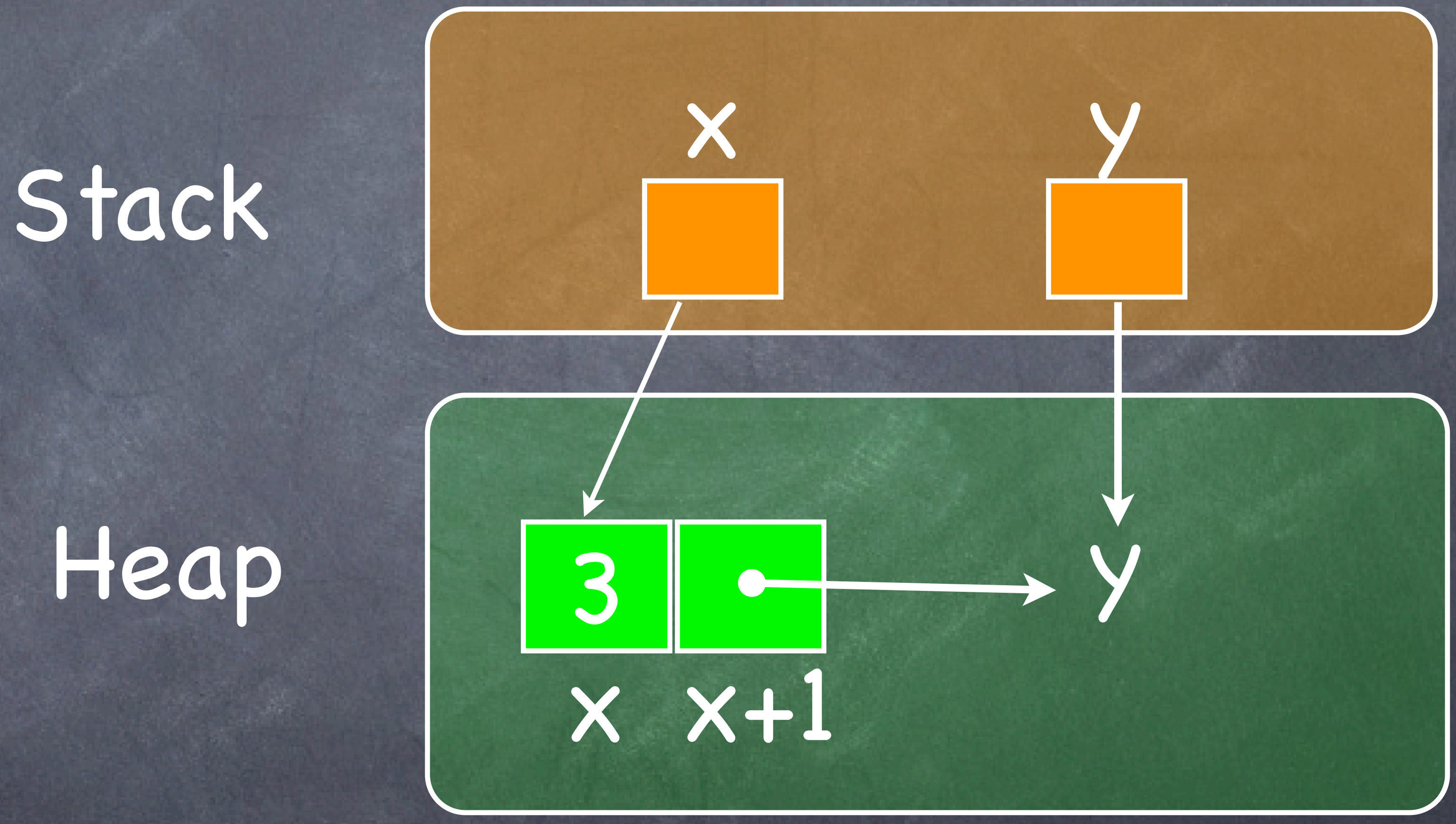

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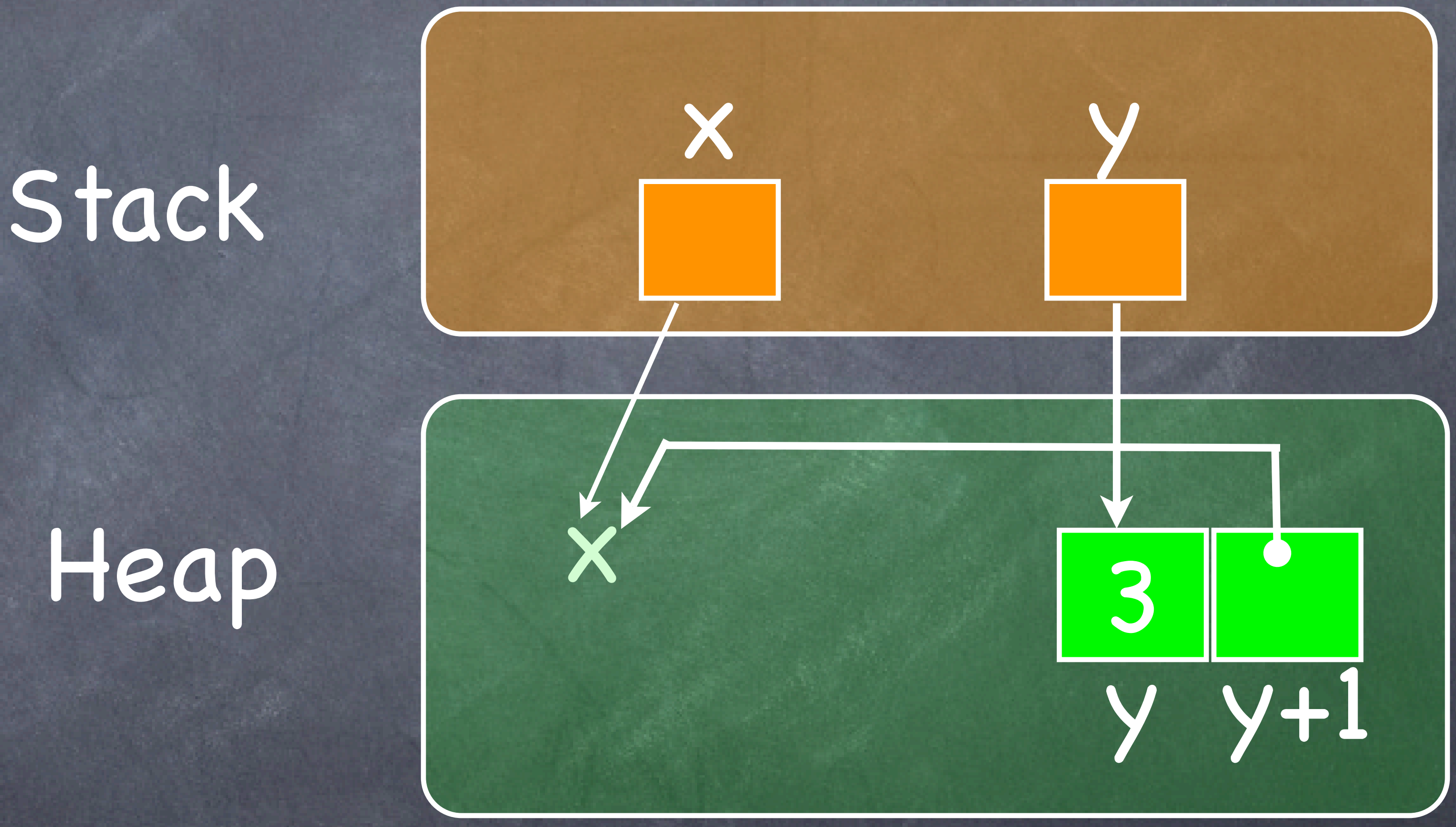

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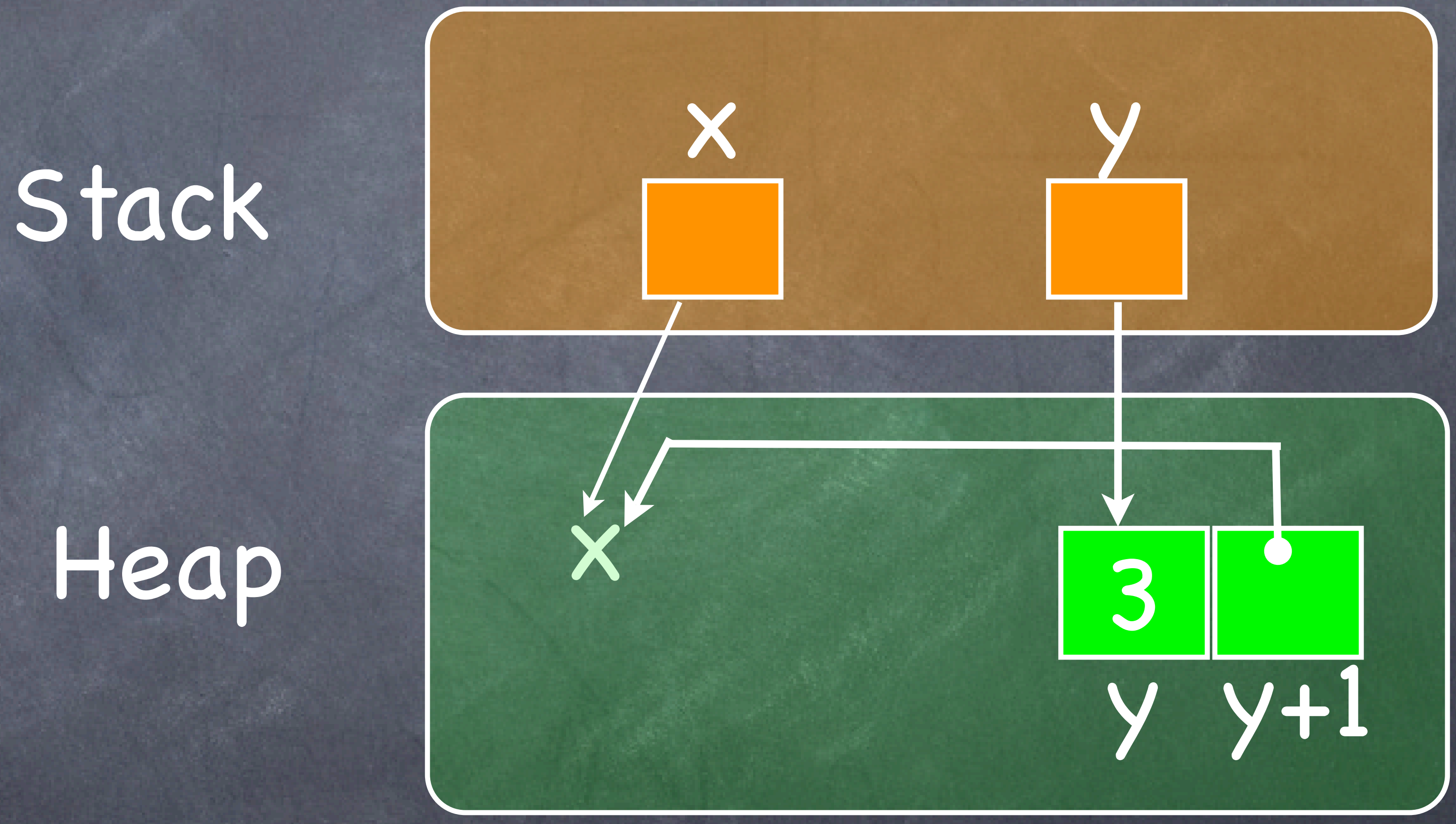
$x \mapsto 3, y * y \mapsto 3, x$



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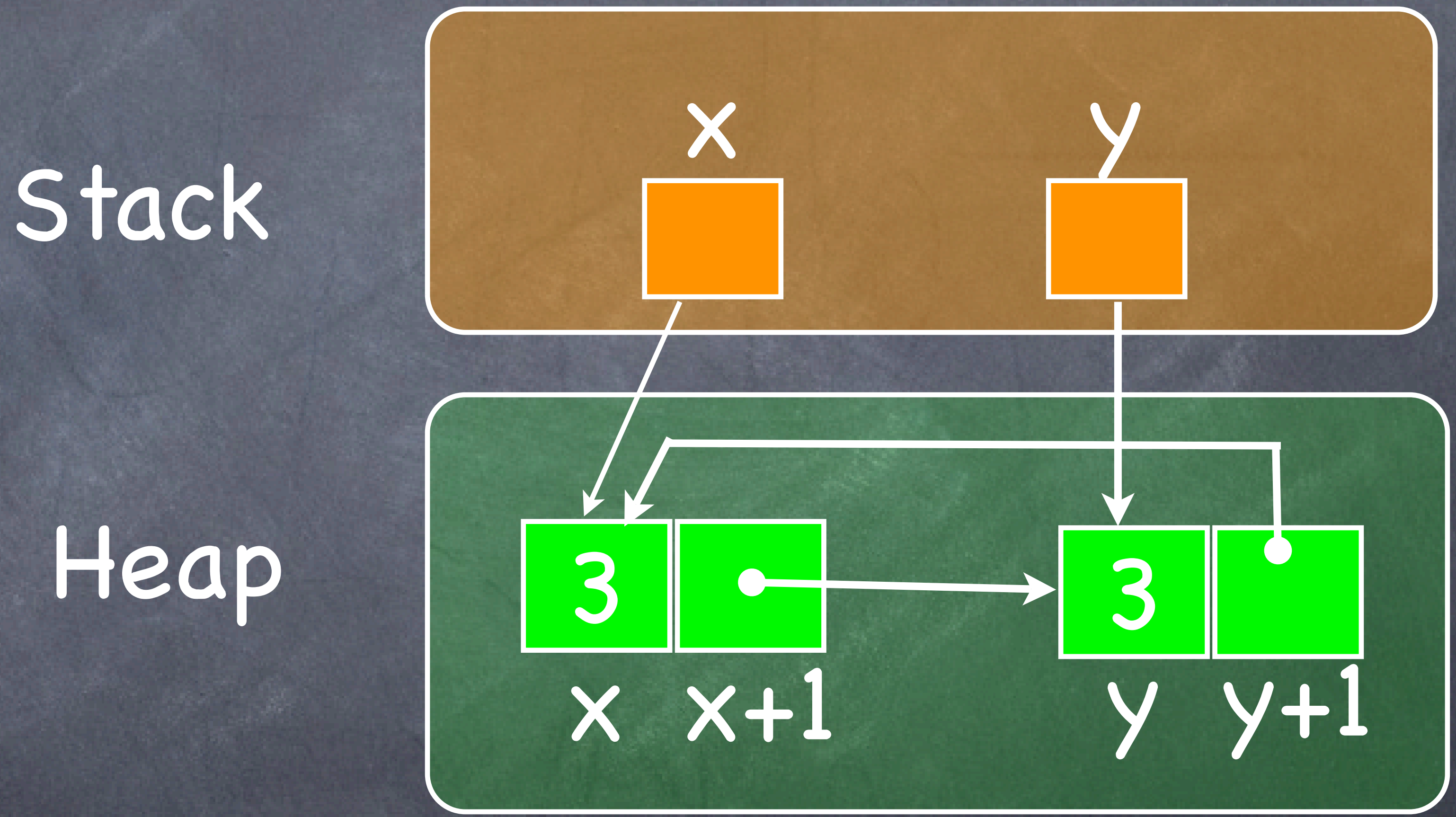

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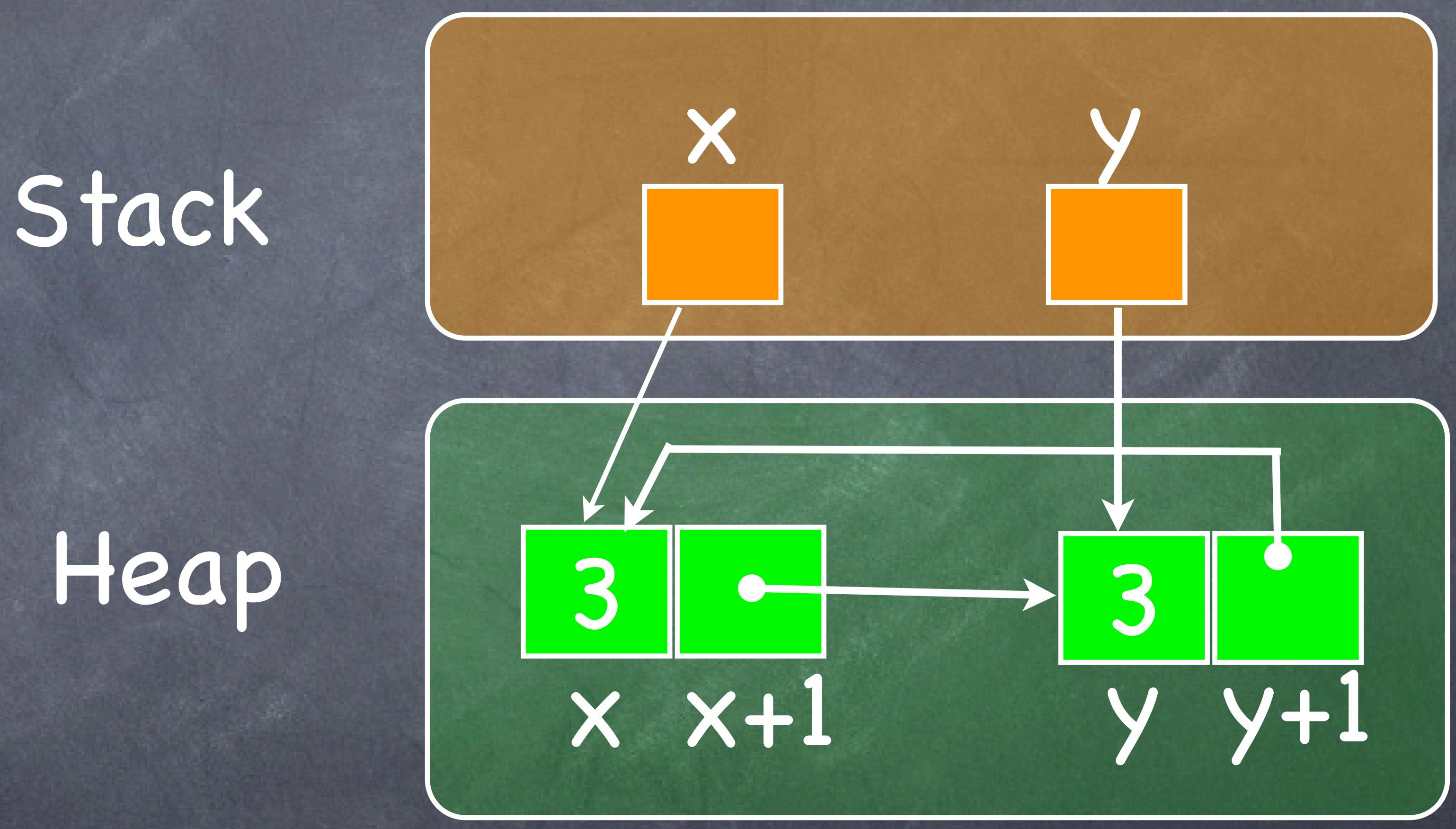

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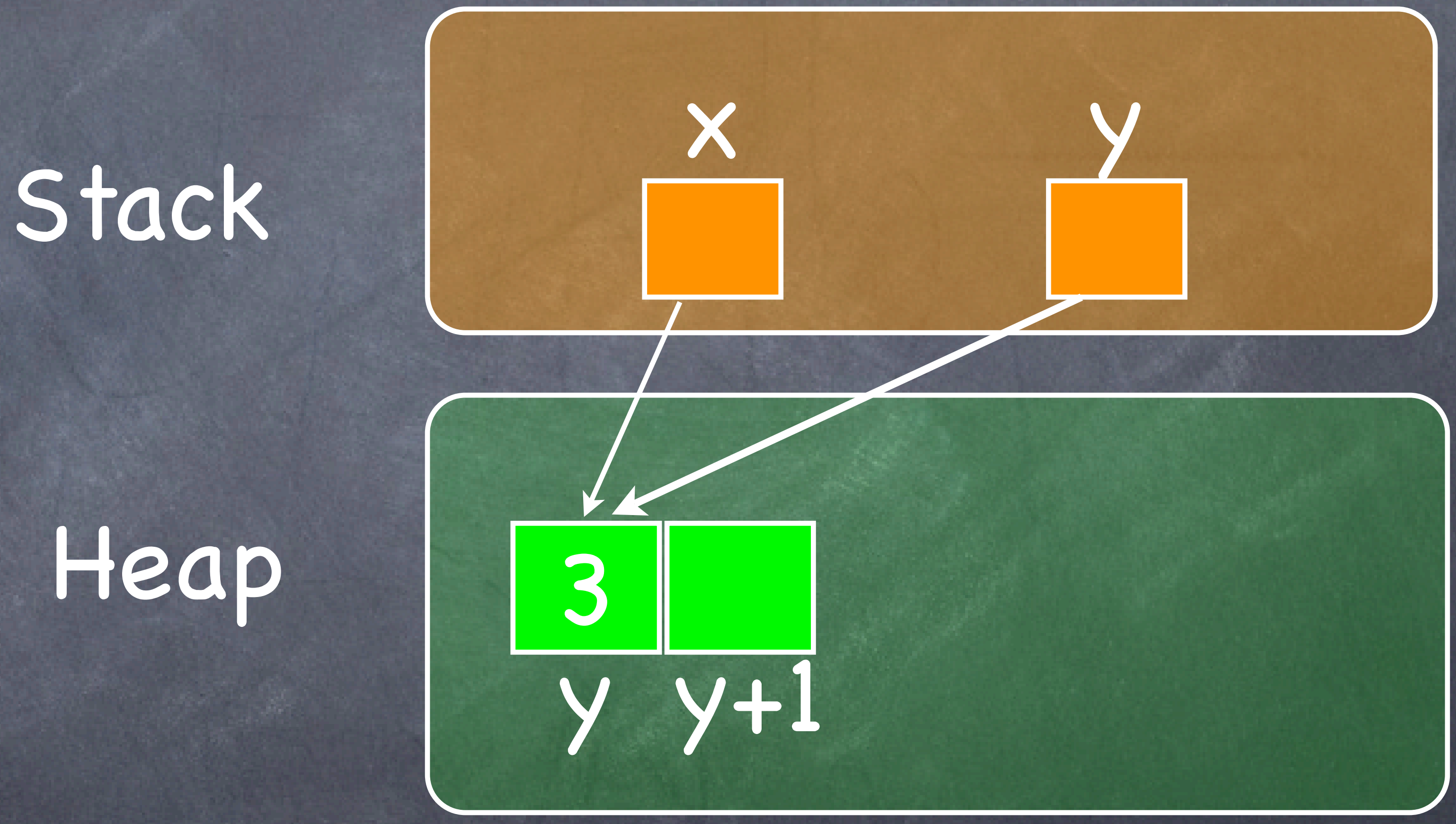

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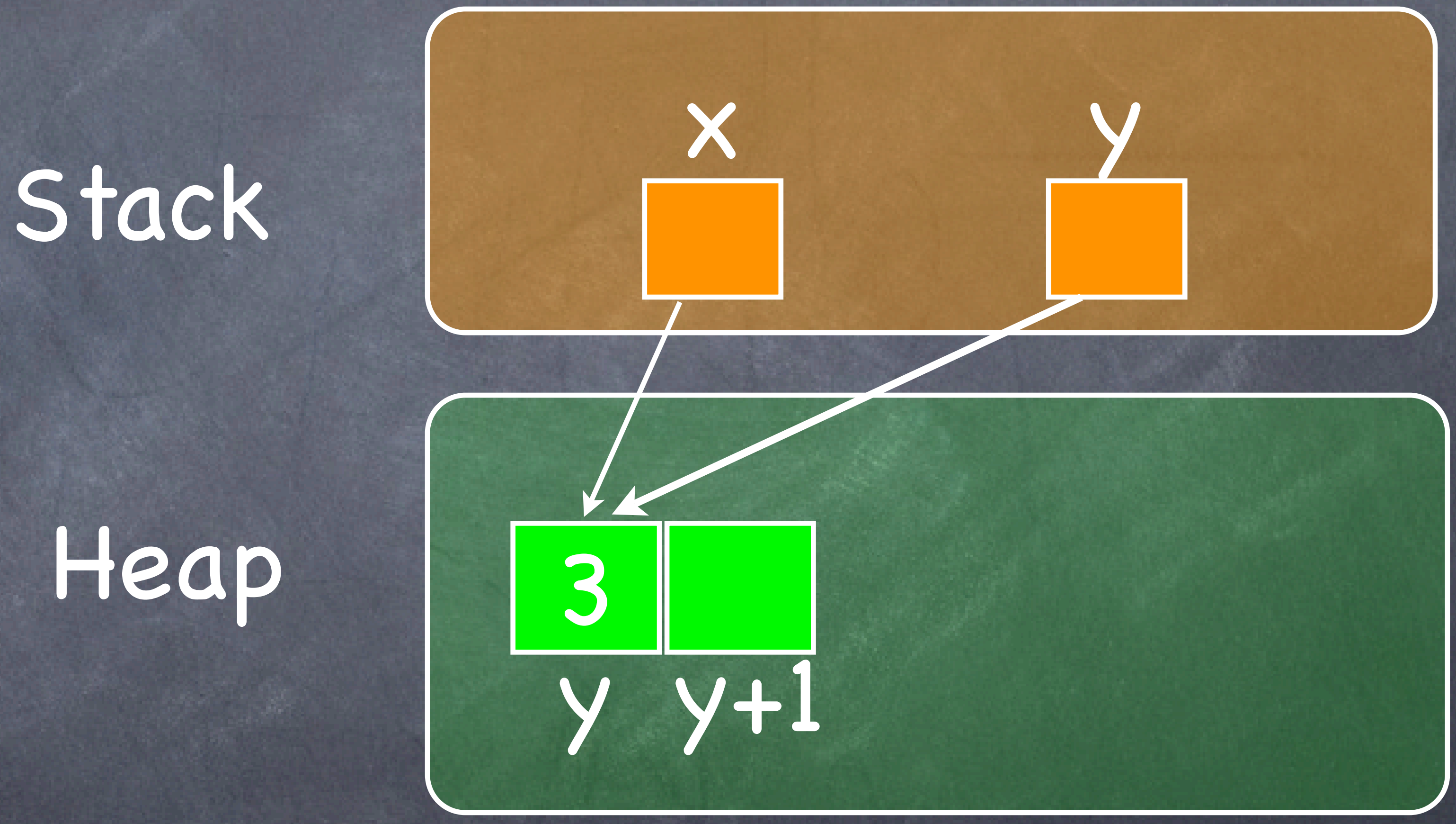

Exercise: what's the last formula asserting?

An inconsistency

- What's wrong with the following formula?
  - 10|->3 * 10|->3

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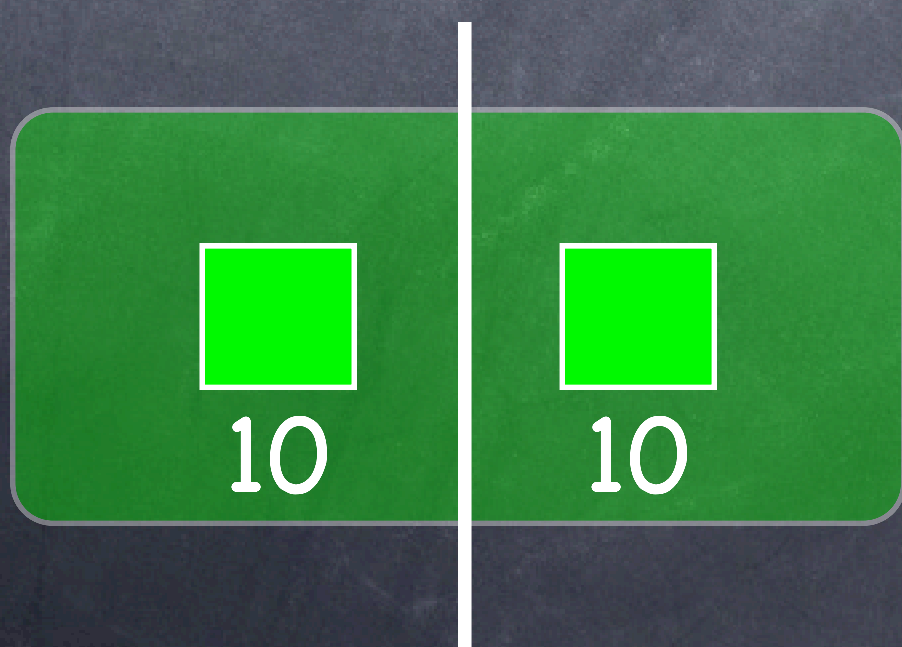

Try to be in two places at the same time

Exercise

what is h such that s,h|= p

h1={(s(x),1)}

h2={(s(y),2)}

with s(x)!=s(y)

Exercise

what is h such that s,h|= p

x|->1

h1={(s(x),1)}

h2={(s(y),2)}

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Exercise

what is h such that s,h|= p

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x|->1

h=h1

Exercise

what is h such that s,h|= p

x|->1

y|->2

h=h1

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x|->1 h=h1

y|->2 h=h2

Exercise

what is h such that s,h|= p

h1={(s(x),1)}

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x|->1 h=h1

y|->2 h=h2

x|->1 * y|->2

Exercise

what is h such that s,h|= p

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x|->1	h=h1
y|->2	h=h2
x|->1 * y|->2	h=h1 * h2

Exercise

what is h such that s,h|= p

h1={(s(x),1)}

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with s(x)!=s(y)

x|->1 h=h1

y|->2 h=h2

x|->1 * y|->2 h=h1 * h2

x|->1 * true

Exercise

what is h such that s,h|= p

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x|->1	h=h1
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x|->1 * y|->2	h=h1 * h2
x|->1 * true	h1 contained in h

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what is h such that s,h|= p

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x|->1 h=h1

y|->2 h=h2

x|->1 * y|->2 h=h1 * h2

x|->1 * true h1 contained in h

x|->1 * y|->2 * (x|->1 \/ y|->2)

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what is h such that s,h|= p

h1={(s(x),1)}

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x|->1	h=h1
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x|->1 * y|->2	h=h1 * h2
x|->1 * true	h1 contained in h
x|->1 * y|->2 * (x|->1 \/ y|->2)	Homework!

Validity

- P is valid if, for all s,h, s,h|=P
- Examples:
  - E|->3 => E>0
  - E|-> - * E|-> -
  - E|-> - * F |-> - => E != F
  - E |-> 3 /\ F |-> 3 => E=F
  - E|->3 * F |->3 => E|->3 /\ F |->3

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  - E |-> 3 /\ F |-> 3 => E=F Valid!
  - E|->3 * F |->3 => E|->3 /\ F |->3 Invalid!

Homework

- Determine/Draw the stack and heap corresponding to the following formulae
  - x|->4,5
  - x|->4,4 * y|->4,4
  - x|->4,4 /\ y|->4,4
- Say if the following are valid or not
  - x|->3 * y|->7 ==> x|->3 * true
  - true * x|->3 ==> x|->3

Some Laws and inference rules

$$
\begin{array}{lcl}
p_1 * p_2 & \iff & p_2 * p_1 \\
(p_1 * p_2) * p3 & \iff & p_1 * (p_2 * p_3) \\
p * \mathsf{emp} & \iff & p \\
(p_1 \vee p_2) * q & \iff & (p_1 * q) \vee (p_2 * q) \\
(\exists x.p_1) * p_2 & \iff & \exists x.(p_1 * p_2) \quad \text{when } x \text{ not in } p_2 \\
(\forall x.p_1) * p_2 & \iff & \forall x.(p_1 * p_2) \quad \text{when } x \text{ not in } p_2
\end{array}
$$

$$
\frac{p_1 \implies p_2 \qquad q_1 \implies q_2}{p_1 * q_1 \implies p_2 * q_2}
$$

Monotonicity

Some Laws and inference rules

...but

$$(p_1 \wedge p_2) * q \implies (p_1 * q) \wedge (p_2 * q)$$

Exercise: prove that the other direction does not hold

$$(\forall x.p_1) * p_2 \iff \forall x.(p_1 * p_2) \quad \text{when } x \text{ not in } p_2$$

$$\frac{p_1 \implies p_2 \qquad q_1 \implies q_2}{p_1 * q_1 \implies p_2 * q_2}$$ Monotonicity

Substructural logic

- Separation logic is a substructural logic:

No Contraction $A \nvdash A * A$

No Weakening $A * B \nvdash A$

Examples:

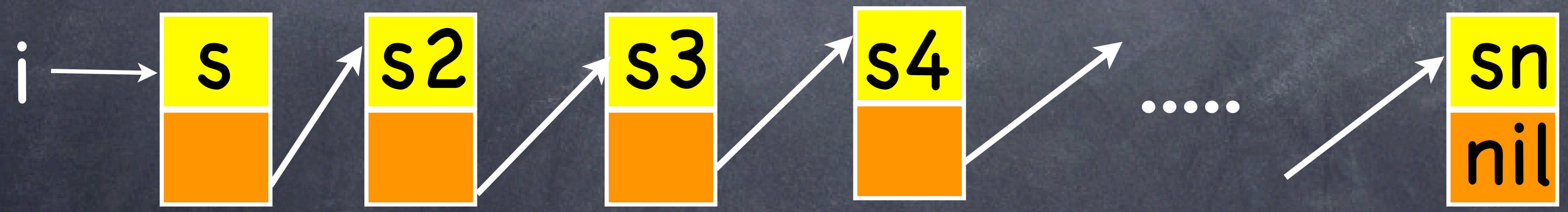
$$10 \mapsto 3 \nvdash 10 \mapsto 3 * 10 \mapsto 3$$

$$10 \mapsto 3 * 42 \mapsto 7 \nvdash 42 \mapsto 7$$

Lists

A non circular list can be defined with the following inductive predicate:

```
    list [] i    = emp /\ i=nil
 list (s::S) i   = exists j. i|->s,j * list S j
```



List segment

Possibly empty list segment

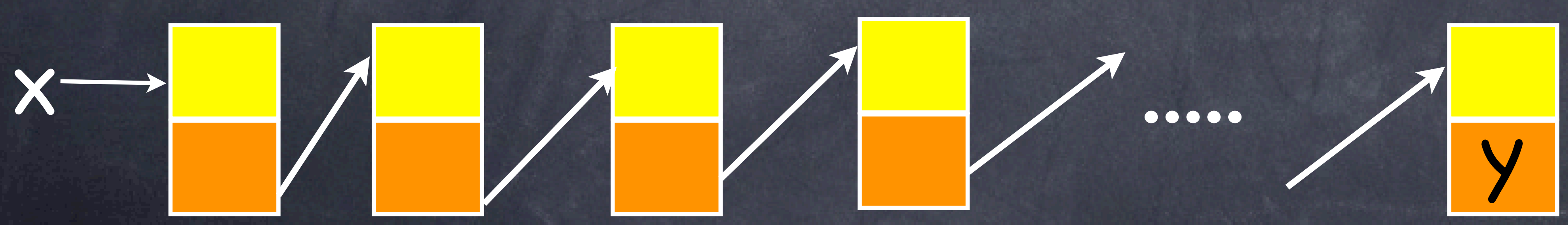
lseg(x,y) = (emp /\ x=y) OR

exists j. x|->j * lseg(j,y)

Non-empty non-circular list segment

lseg(x,y) = x!=y /\

((x|->y) OR exists j. x|->j * lseg(j,y))

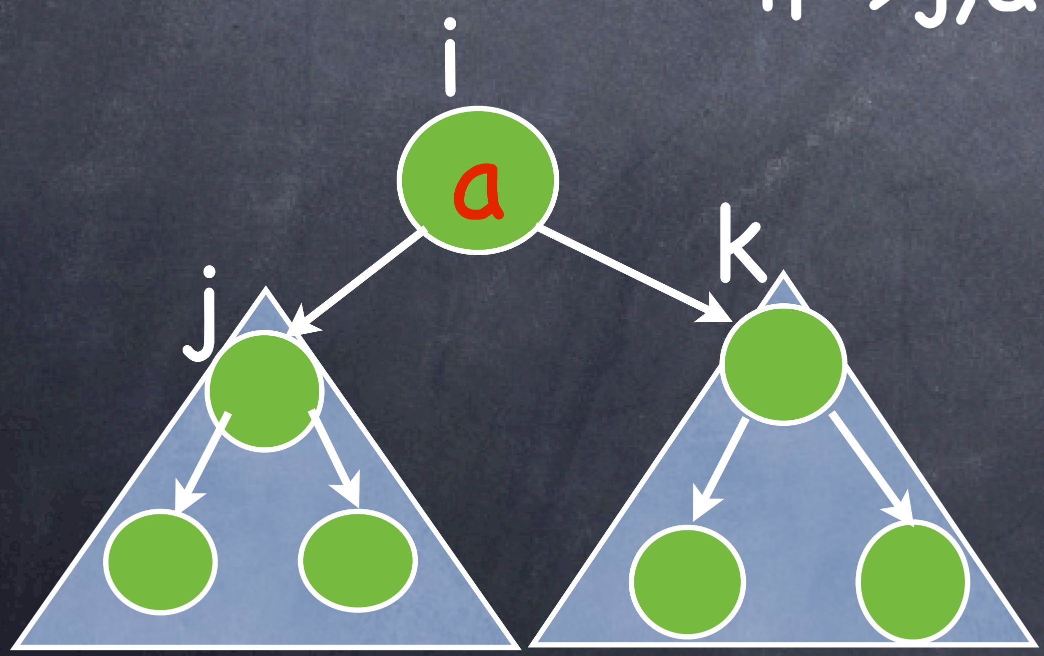


Trees

A tree can be defined with this inductive definition:

tree [] i = emp /\ i=nil

tree (t1,a,t2) i = exists j,k.
i|->j,a,k * (tree t1 j) * (tree t2 k)



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