Markov Analysis of the PRMA Protocol for Local Wireless Networks

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AdHoc Networking SS2004 – Models and Methods

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Motivation

AdHoc Networking

- Interest of mobile tools for personal computing and communication
- Efficient handling of real-time and non-real-time traffic
- Limitations in the available radio spectrum (little up to no central coordination)
- Micro cellular networks meet these conditions due to higher frequency reuse but in turn increase the complexity of these systems

Overview

- Introduction
- The Packet Reservation Multiple Access Protocol
- The Voice Model
- The PRMA Model
- (Packet Dropping Analysis)
- Stability Analysis
- Numerical Examples

Introduction

AdHoc Networking

- PRMA – Packet Reservation Multiple Access
- Contention-based channel access protocol for wireless communication
- Transmitting packetized information over a shared channel

The PRMA Protocol

AdHoc Networking

- Is slotted like the R-ALOHA (reservation-ALOHA), slots are grouped into frames
- Designed for wireless micro cellular networks
- Can handle real-time and non-real-time traffic

Focus here: real-time traffic

The PRMA Protocol

AdHoc Networking

- Terminals either send packets during talkspurts or sleep during silent periods
- As soon the talkspurt starts, it contends with other terminals for unreserved slot
- A contending terminals transmits a packet, if is obtains permission
- Permissions occur with fixed probility at each unreserved time slot, independently at each terminal

The PRMA Protocol

- If two or more contending terminals attempt to send in the same unreserved slot, collision occurs (retransmission)
- Packets delayed beyond D_{max} are dropped by terminals
- If a talk spurt ends before a reservation has been obtained, all remaining packets in the buffer are dropped

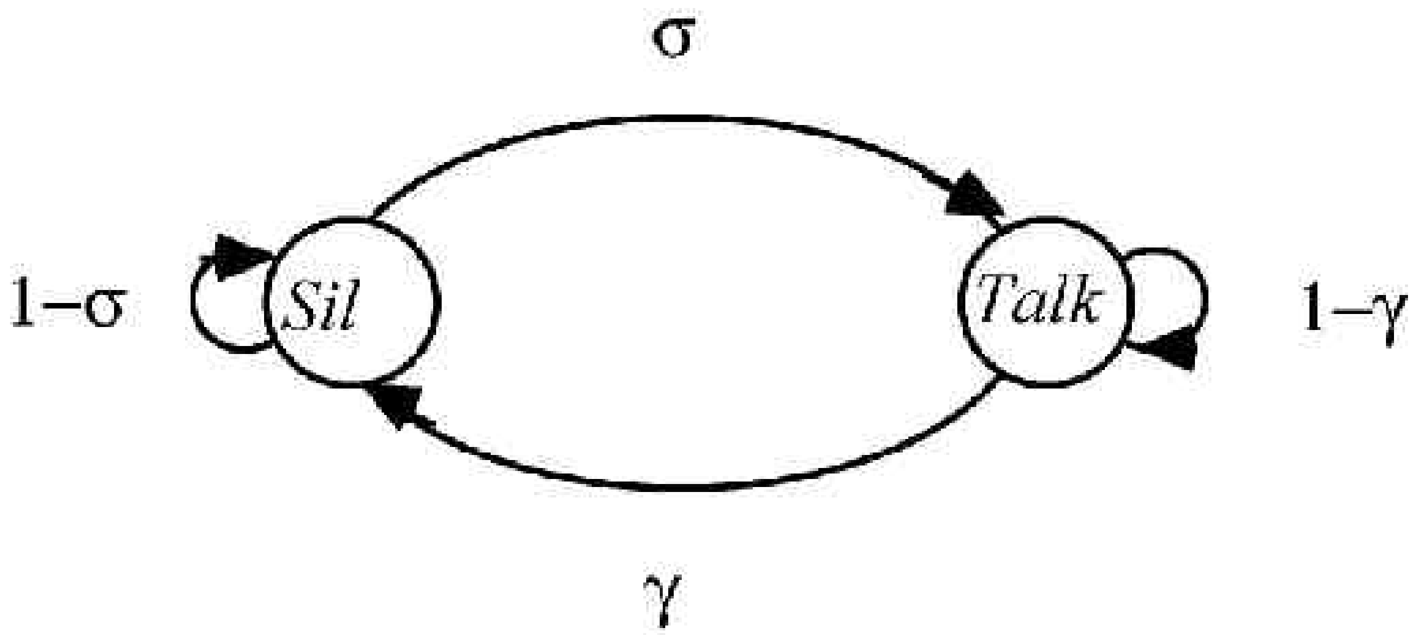
The Voice Model

- Model for voice source is provided by 2 state Markov process:
  - exp distributed talking (active)
  - exp distributed silence (idle)
- t_1 mean length of talking
- t_2 mean length of silence
- τ slot duration

The Voice Model

AdHoc Networking

- γ = Pr{talkspurt ends with mean t_1} = 1-exp($-\tau/t_1$)
- σ = Pr{silence ends with mean t_2} = 1-exp($-\tau/t_2$)
- Talking periods are geom. distributed with mean $1/\gamma$
- Silence periods are geom. distributed with mean $1/\sigma$



- Fraction of time spend in each of the states is thus

$$\pi_{\text{Sil}} = \frac{\gamma}{\sigma + \gamma}$$

and

$$\pi_{\text{Talk}} = \frac{\sigma}{\sigma + \gamma}$$

The PRMA Model

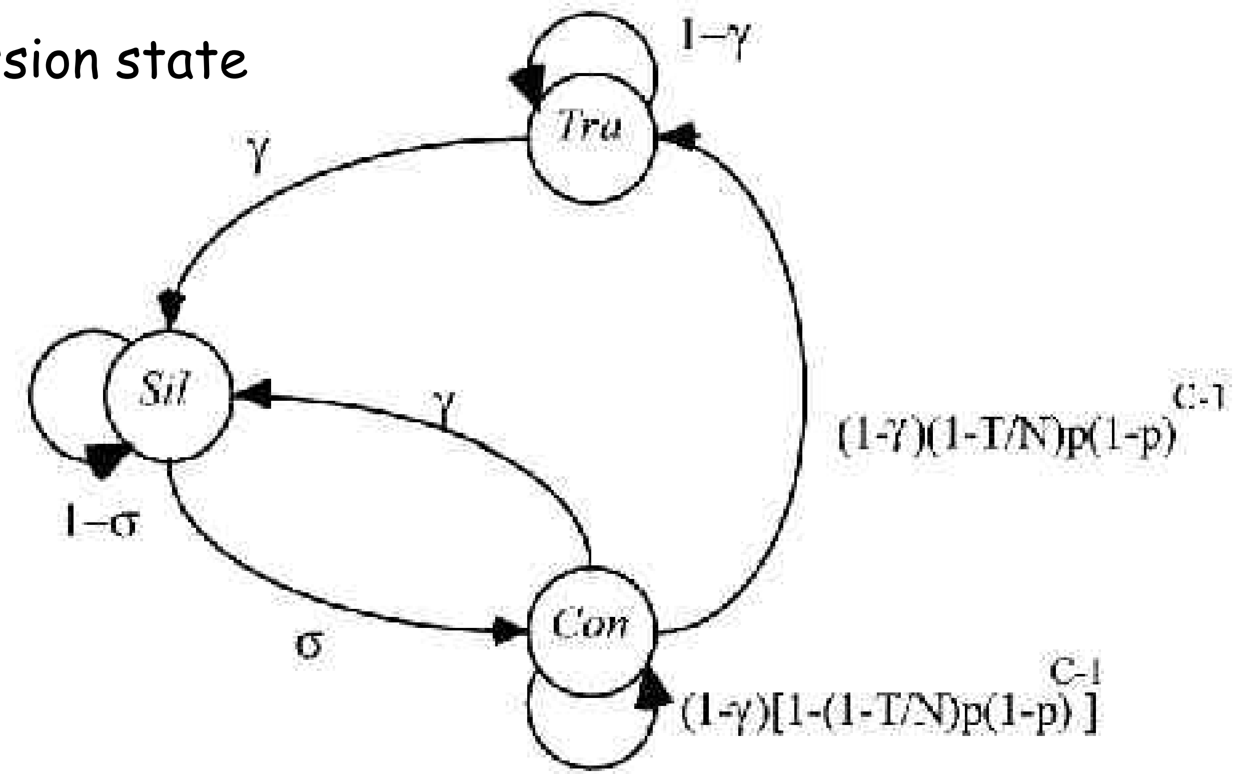
- M homogeneous independent voice terminals
- N number of slots per frame
- p permission to send probability (constant and equal for all terminals)

The PRMA Model

AdHoc Networking

Each terminal is always in on the following states:

- Sil silent state
- Con contending state
- Tra transmission state



The PRMA Model

- Pr{talkspurt does not end} = $(1 - \gamma)$
- Pr{permission to transmit} = p
- Pr{no collision} = $(1-p)^{C-1}$ (C contending terminals)
- For simplicity: Probability of an available time slot is given by the fraction of free time slots

  T denotes number of terminals currently in transmission:

  Pr{available time slot} = $(N-T)/N = 1 - T/N$

The PRMA Model

AdHoc Networking

Pr{transition from Con to Tra} =

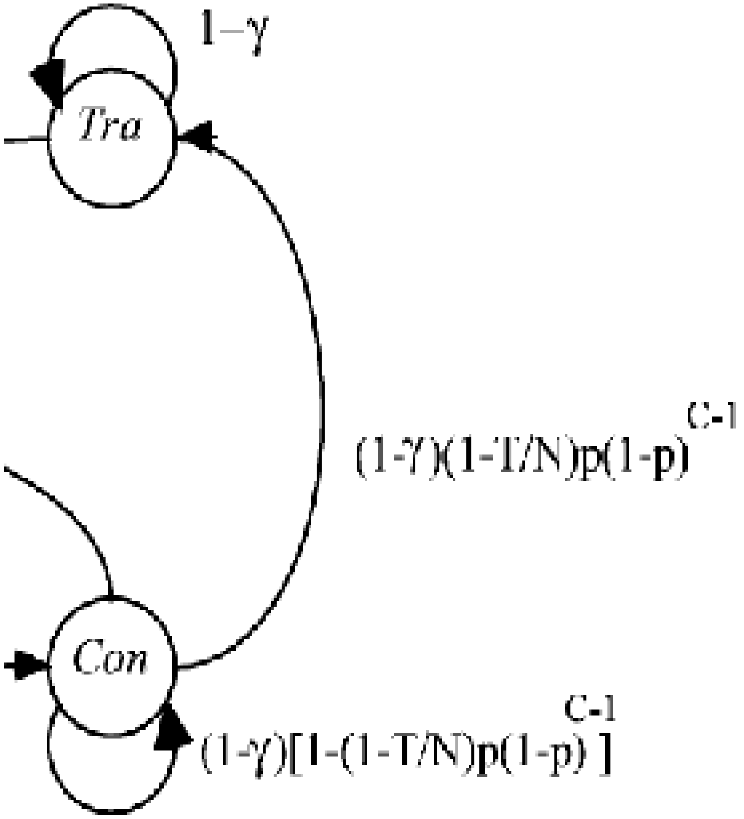
Pr{talkspurt does not end} *

Pr{available time slot} *

Pr{permission to transmit} *

Pr{no collision} =

$(1 - \gamma)\,(1 - T/N)\,p\,(1-p)^{C-1}$



Discrete Time Markov Model

Model of the PRMA voice system as a discrete time Markov process

$$X = \{ X_n = (S_n, C_n, T_n) \mid n >= 0\}$$

with state space

$$\Omega = \{ (s,c,t) \mid s,c,t >= 0,\ s <= M,\ t <= N,\ c = M-t-s \}$$

and transition probability matrix P.

Number of states in Ω is

$$(N + 1)(M - N/2 + 1)$$

Discrete Time Markov Model

- Pr{i transmitting terminals exit to silent state}

$$= \binom{t}{i} \gamma^{i} (1-\gamma)^{t-i} \qquad (1)$$

- Pr{j silent terminals begin to contend}

$$= \binom{s}{j} \sigma^{j} (1-\sigma)^{s-j} \qquad (2)$$

Discrete Time Markov Model

- Pr{k contending terminals return to silent state and h terminals get a reservation and begin transmission} =

$$\binom{c}{k}\gamma(1-\gamma)^{c-k} \times \begin{cases} 1-(1-\frac{t}{N})(c-k)p(1-p)^{c-k-1}, h=0 \\ (1-\frac{t}{N})(c-k)p(1-p)^{c-k-1}, h=1 \end{cases}$$

(3)

Discrete Time Markov Model

Entries of one-step probability matrix P with (1), (2), (3) are

$$\Pr\{ X_{n+1} = (s',c',t') \mid x_n = (s,c,t) \} =$$

$$\sum_{\substack{s+i-j+k=s' \\ c+j-k-h=c' \\ t-i+h=t'}} \alpha_{ijkh} \quad \text{with} \quad \alpha_{ijkh} =$$

Pr {i transmitting terminals to exit to silent state} x

Pr{j silent terminals begin to content} x

Pr{k contending terminals return to silent state and h terminals get a reservation and begin transmission}

Discrete Time Markov Model

- stationary probability distribution:

$$\pi = [\pi_{(s,c,t)}](s,c,t) \in \Omega$$

- From the stationary distribution vector π the stationary distribution of the system variables S, C and T (number of terminals in each of the states) can be computed.

Discrete Time Markov Model

AdHoc Networking

- Stationary distribution of the system variables S, C and T (number of terminals in each of the states)

$$p_S(k)=\Pr\{S=k\}=\sum_{(s,c,t)\in\Omega,s=k}\pi_{(s,c,t)}$$

$$p_C(k)=\Pr\{C=k\}=\sum_{(s,c,t)\in\Omega,c=k}\pi_{(s,c,t)}$$

$$p_T(k)=\Pr\{T=k\}=\sum_{(s,c,t)\in\Omega,t=k}\pi_{(s,c,t)}$$

for k=0, ..., M

Expected values:

$$E[S] = \sum_{k=0}^{M} k p_S(k)$$

$$E[C] = \sum_{k=0}^{M} k p_C(k)$$

Throughput: average number of transmitted packets per frame

$$E[T] = \sum_{k=0}^{N} k p_T(k)$$

Throughput, Utilization, Access Delay

AdHoc Networking

- Utilization: fraction of slots per frame used to transmit packets

$$E[T]/N$$

- Using Little's Law : $E[W]=E[N_q]/\lambda$

Avg no of customers (N_q)

Avg waiting time ($E[W]$)

Avg arrivals per time unit (λ)

- the access delay W is

$$E[W]=\frac{E[C]}{E[S]\cdot\sigma}=\frac{E[C]}{\sigma}\cdot\frac{\gamma+\sigma}{M\gamma}$$

Stability Analysis

AdHoc Networking

- Markovian Model equilibrium point defined as:

  Values of the state variables for which the expected change in each state variable equals zero.

- Let s, c and t be the equilibrium point number of terminals in silence, contention, and transmission respectively.

- Equilibrium at state Tra $(1-\gamma)\left(1-\frac{t}{N}\right)c \cdot p \cdot u(c) = t\gamma$

Inflow

Outflow

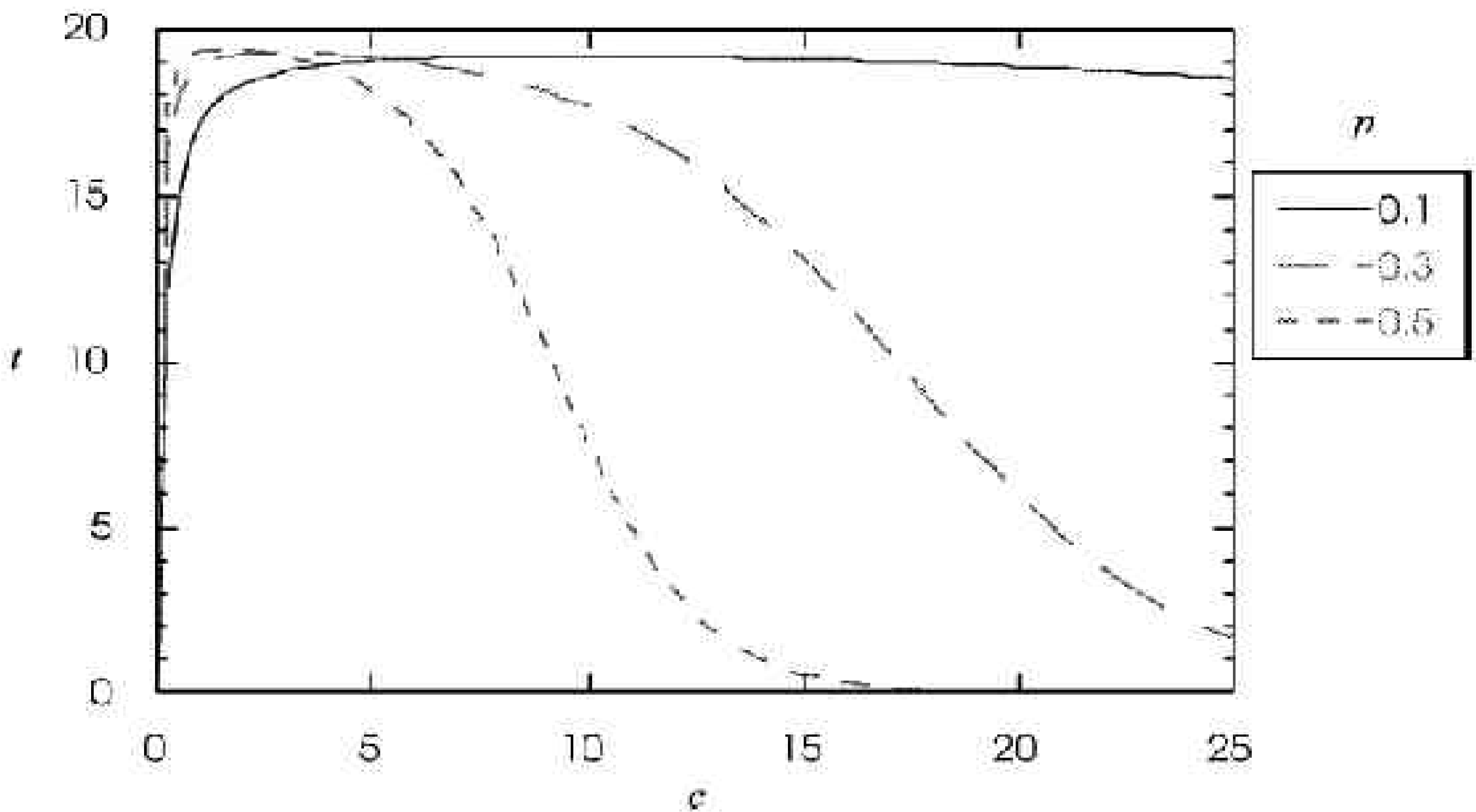
with $$u(c) = \begin{cases} 1 & , c = 0 \\ (1-p)^{c-1} & , c \geq 1 \end{cases}$$

Stability Analysis - EQ-contour

AdHoc Networking

- Equilibrium Contour in the (c,t)-plane:

δ(contending terms begin trans.) = δ(transmitting terms end trans.)



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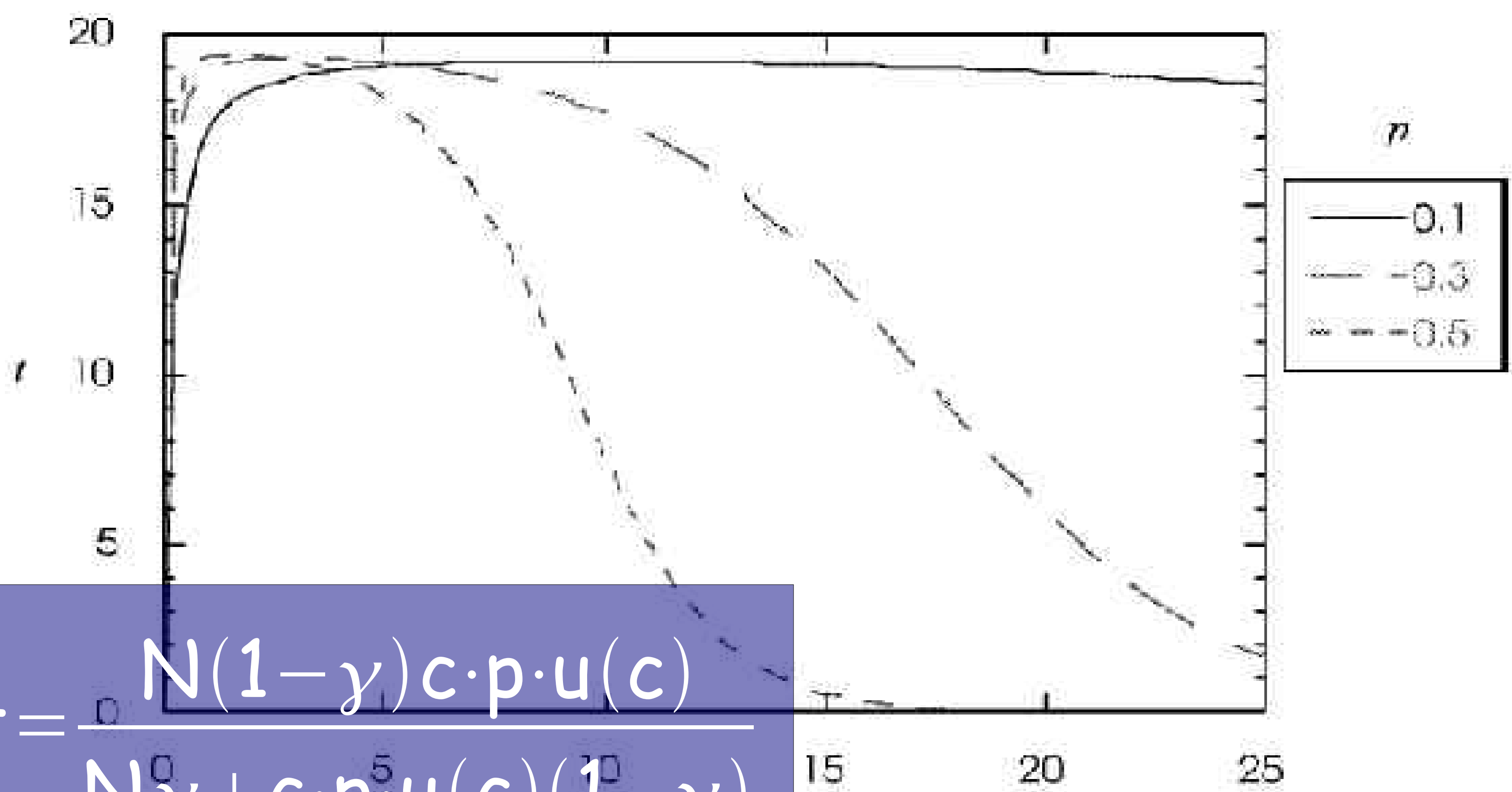
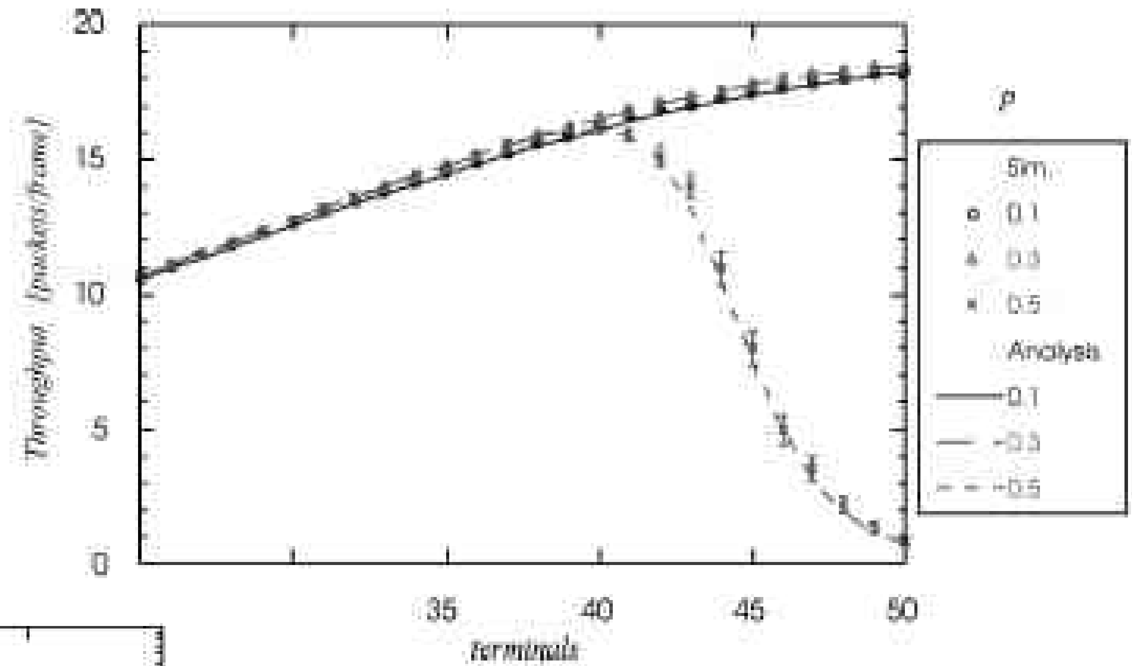
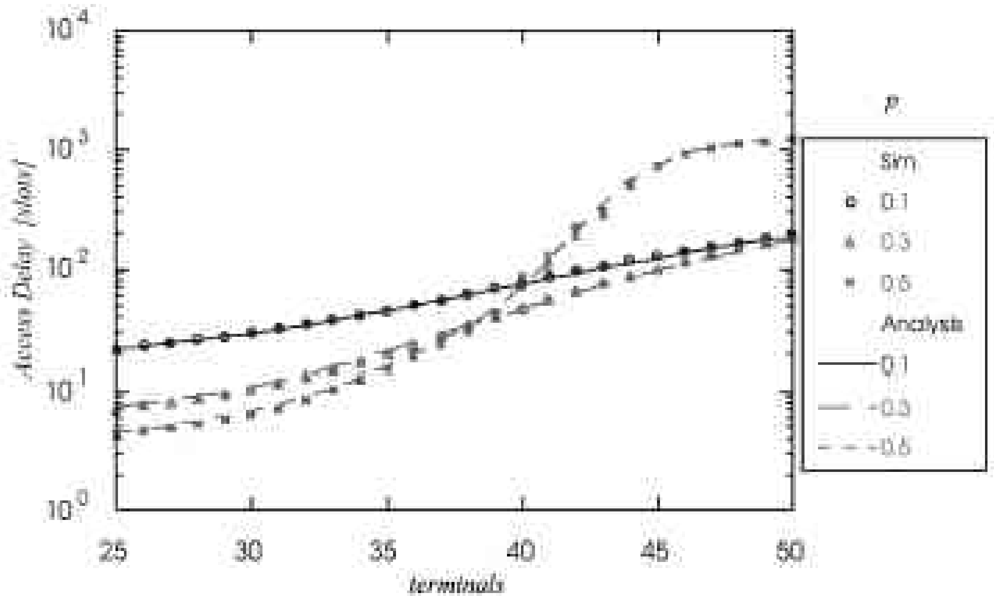


$$t=\frac{N(1-\gamma)c\cdot p\cdot u(c)}{N\gamma+c\cdot p\cdot u(c)(1-\gamma)}$$

Numerical Examples

AdHoc Networking





Frame duration: 16ms

No of slots/frame (N): 20

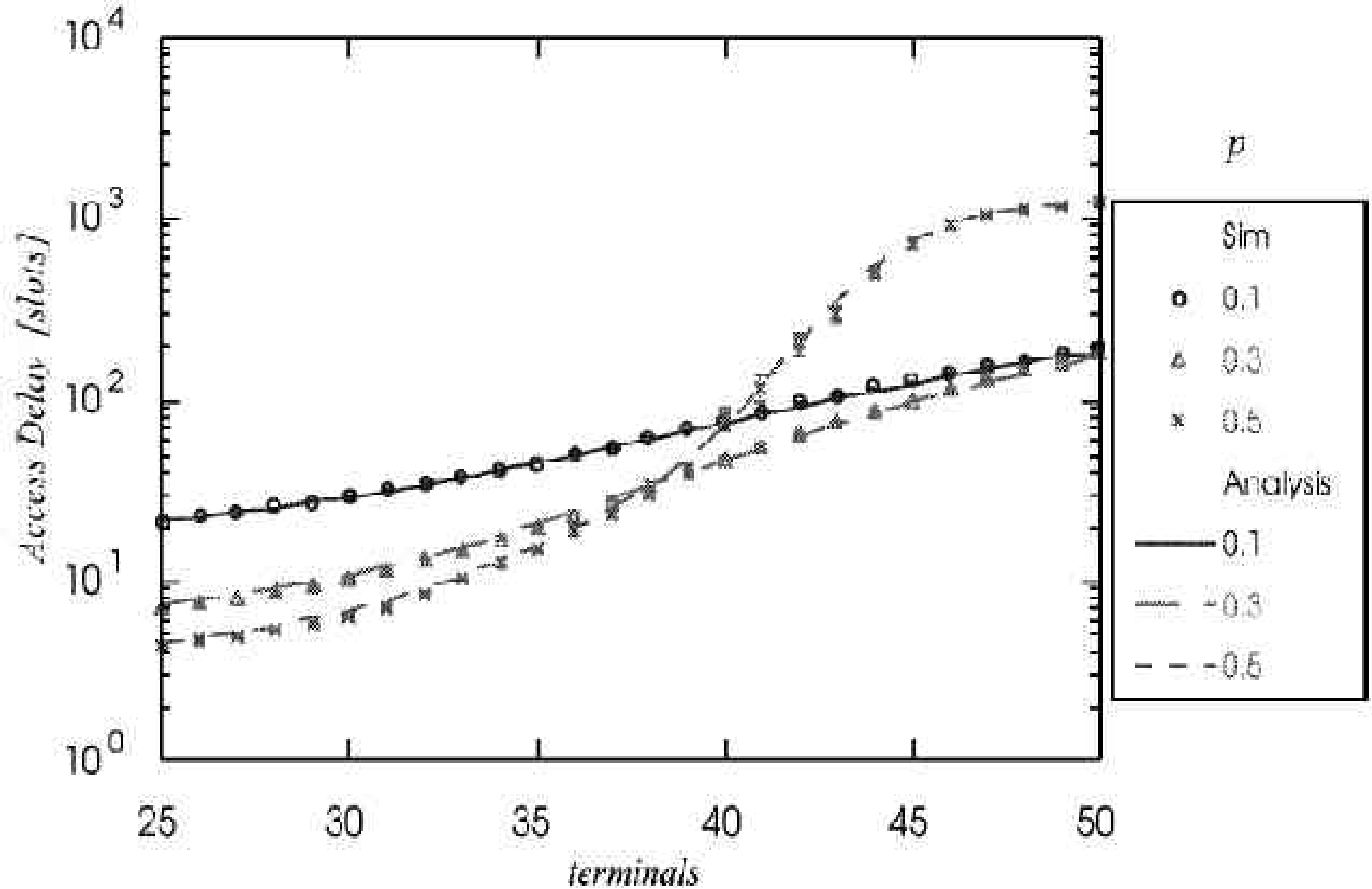
Maximum holding time: 32ms

D_{max}: 40 slots

PRMA protocol for local wireless networks

Numerical Examples

AdHoc Networking



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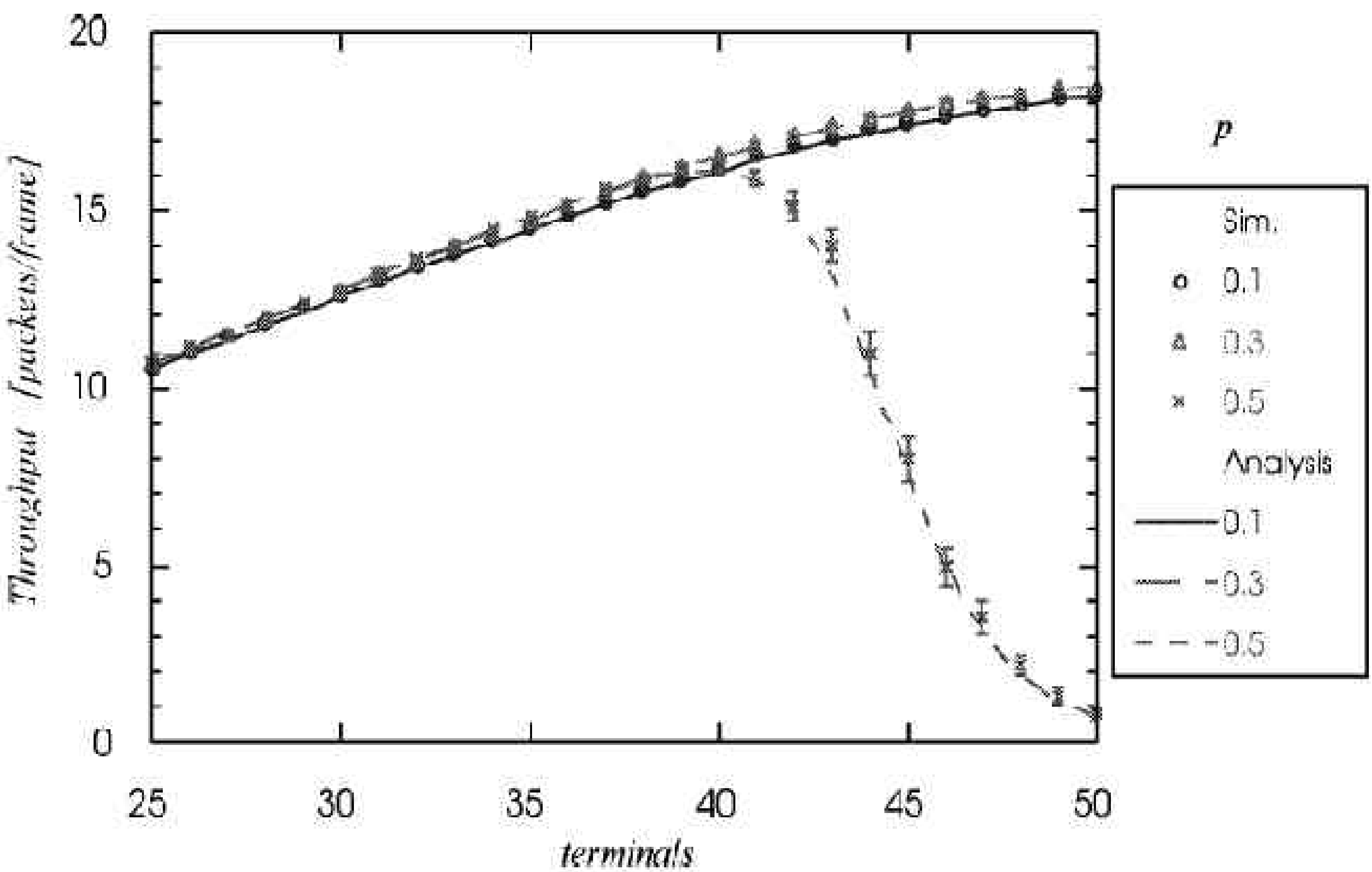
Numerical Examples

AdHoc Networking

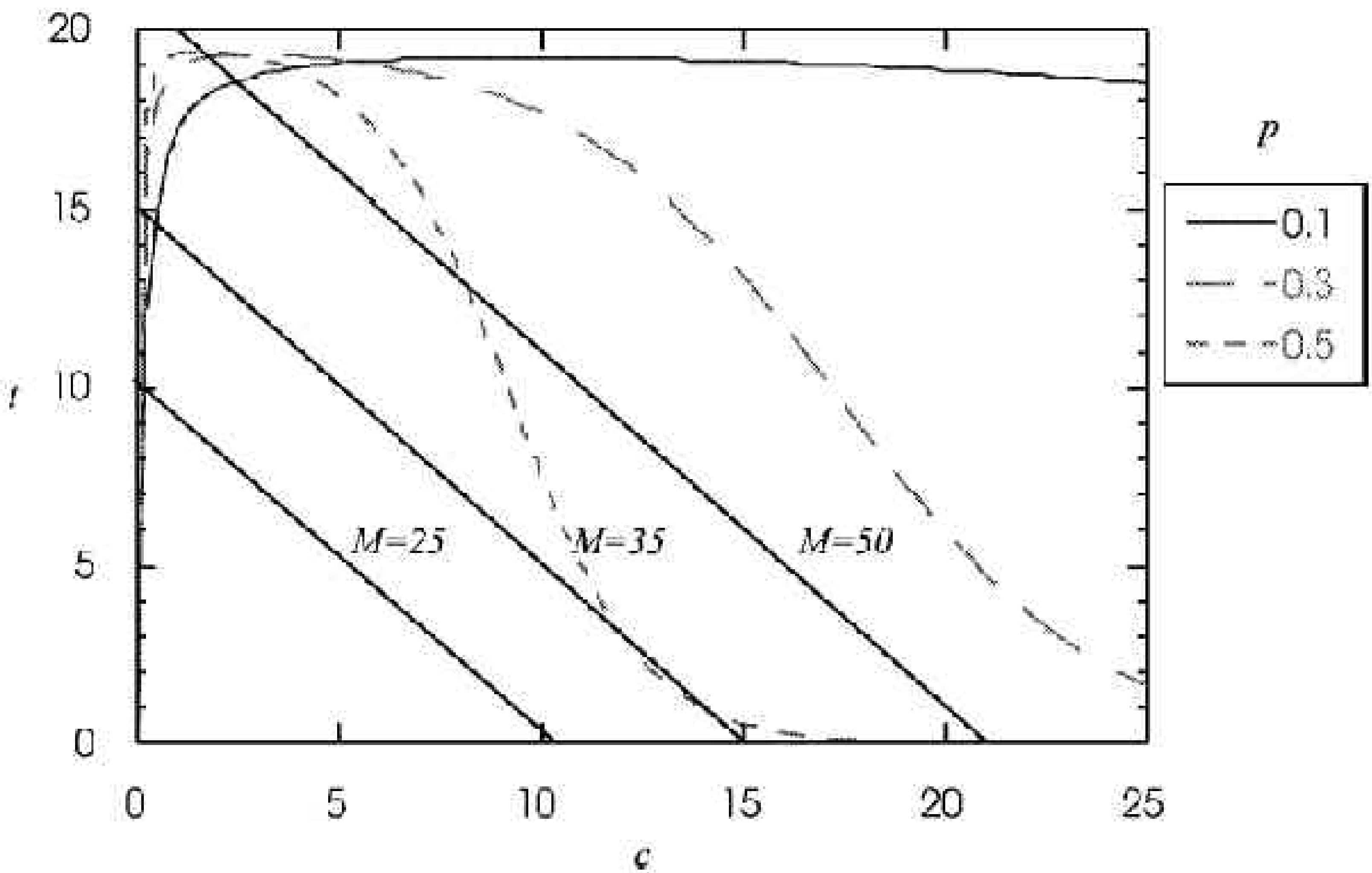


Markov analysis of the PRMA protocol for local wireless networks

Numerical Examples

AdHoc Networking

PRMA system stability analysis



Conclusion

AdHoc Networking

- Within a talkspurt, several consecutive packets can be lost with serve performance degradation
- Packet dropping distribution provides a better characterization of the quality perceived by the user
- Probability p of obtaining permission to transmit greatly influences the quality of service
- Hmmm... what about packet dropping analysis???

END