

(J.Q. Bao, L. Tong)

# A Performance Comparison between Ad Hoc and Centrally Controlled CDMA Wireless LANs

Ad Hoc Networking  
- Prof. Hermanns -  
SS 2004

Stephan Zimmer

# Outline

- I. Introduction
- II. Two System Models
- III. Markovian Analysis
- IV. Performance Comparison
- v. Conclusion

# Introduction

*Goal:*

Performance comparison between two types of code division multiple-access wireless LANs:

**Centrally Controlled**

and

**Ad Hoc**

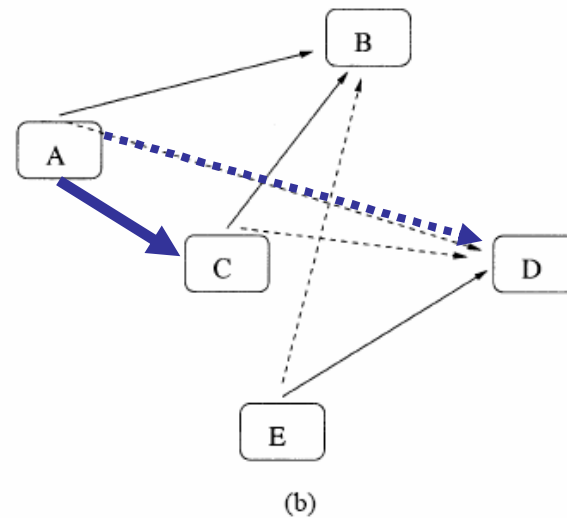
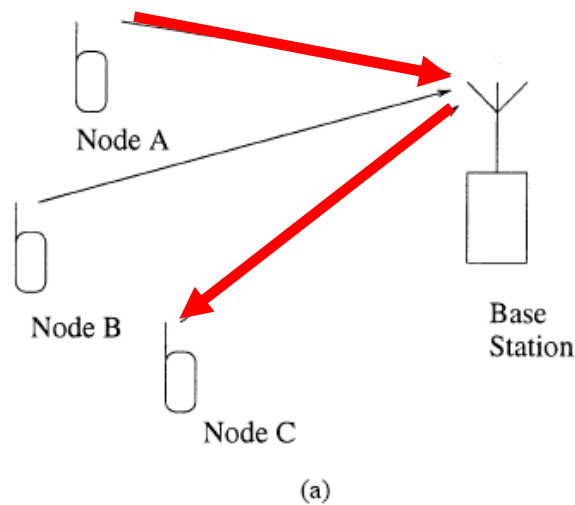


Fig. 1. (a) Centrally controlled system. (b) Ad hoc system.

# Introduction

## General restrictions:

- packet-switched code division multiple-access (CDMA) system
- employing *slotted ALOHA* random access protocols
- restriction to small coverage area allows to ignore near-far effect

# System Models

## Description Centrally Controlled Network:

Multiple nodes transmit packets to each other through the *Base Station (BS)*.

Time division duplex system (TDD) with equal sized uplink and downlink packets, each occupies one **time slot**.

Nodes: half-duplex and always in receiving mode during downlink and in transmitting mode during uplink.

*Slotted ALOHA* random access protocol is used by all nodes in the uplink.

Each node uses a unique spreading code in the uplink.

Receiver at the Base Station is a bank of *matched filters*.

Base Station uses orthogonal codes (for packets intended for different receivers)

-> receiving node is always successful

-> transmission only depends on uplink reception

# System Models

## Description Ad Hoc System:

Nodes transmit **directly** to each other (through a common channel).

Each node can be transmitter or receiver.

Also slotted ALOHA is employed by all nodes.

Every node uses a unique code to transmit packets.

(Assume each node has knowledge of all possible spreading codes)

Different *reception capability* of nodes compared to CC system:

i.e. a transmitting node in the ad hoc network cannot receive packets from other nodes

-> collision between transmitter and receiver

# System Models

## Assumptions and notations:

*Backlogged node* : a node which needs to retransmit a packet  
(otherwise the node is the *unbacklogged* state)

*Finite-population model* is used  
(Number of nodes in both systems finite.)

Ignore channel noise and assume that errors in a packet are caused by  
multiple access interference (MAI) alone.

## Important used parameters:

M : total number of nodes

n : number of backlogged nodes

N : spreading gain

t : number of correctable errors

# System Models

**Assumption 1:** Nodes generate packets according to independent Poisson processes with equal arrival rate  $\lambda$

**Assumption 2:** Immediate feedback about the status of transmission

**Assumption 3:** No buffer at any node  
(i.e. each node can at most hold one packet at a time)

**Assumption 4:** With probability  $s_{ki}$  the receiver (in both systems) successfully detects  $i$  out of  $k$  colliding packets in a time slot

**Assumption 5:** Each node has equal probability to transmit to every other node



# Markovian Analysis

For performance evaluation we need to set up a mathematical model for these two systems

*Markov chain* approach for finite-population model  
with  $n$  = number of backlogged nodes as network state

-> for  $M$ -node network:

$$(M+1) \times (M+1) \text{ **transition matrix** } P = [p_{nk}]$$

$p_{nk}$  = probability that network state goes from  $n$  to  $k$  in one step

# Markovian Analysis

- Centrally Controlled Network -

*The goal:*

transition matrix  $P^c = [p_{nk}^c]$  for centrally controlled network

$p_{nk}^c$  = probability that network state goes from  $n$  to  $k$   
in **two** time slots

To obtain  $P^c$  we need:

- *reception matrix*  $S = [s_{jk}]$

$$S = \begin{pmatrix} s_{10} & s_{11} & 0 & \dots & 0 \\ s_{20} & s_{21} & s_{22} & \dots & 0 \\ \dots & & & & \dots \\ \dots & & & s_{(M-1)(M-1)} & 0 \\ s_{M1} & s_{M2} & s_{M3} & \dots & s_{MM} \end{pmatrix}$$

with  $s_{jk}$  : pr. that the BS demodulates  $k$  out of  $j$  packets

# Markovian Analysis

## - Centrally Controlled Network -

Elements of S are a function of the pr. of a node to *successfully detect a packet in a collision*

For CDMA packet systems it is *difficult* to determine the exact probability  $p_c(k)$  of successfully detecting a packet by a receiver

*Additional simplification:*

Output of matched filter corresponding to multiple access interference (MAI) assumed to be a white Gaussian random process

Leads to packet success probability (see [6]):

$$p_c(k) = \sum_{i=0}^t \binom{L}{i} x^i (1-x)^{L_p-i}$$

With parameters:

t : number of bit errors that can be corrected by coding

L : length of packet (bits)

x : bit error rate (BER)


$$x = Q\left(\sqrt{\frac{3N}{k-1}}\right)$$

# Markovian Analysis

- Centrally Controlled Network -

Each matched filter at BS 's receiver works independently:

Therefore  $s_{nk}$  = pr. that n out of k independent Bernoulli trials are successful  
with a single trial success probability  $p_c(k)$

$$s_{nk} = \binom{k}{n} p_c(k)^n (1 - p_c(k))^{k-n}$$


-> elements of the reception matrix S

*Remind:* We (still) want the **transition matrix  $P^c$**

What else is needed to determine  $P^c$ ?

How is change of network state defined?

# Markovian Analysis

## - Centrally Controlled Network -

*Change of network state* is determined by  
difference between *unsuccessful* transmissions from *unbacklogged nodes*  
and the *successful* transmission from *backlogged nodes*

In addition to the reception matrix S we need:

pr. that k *unbacklogged* nodes transmit  $Q_a^c(k, n) = \binom{M-n}{k} (1-p_a^c)^{M-n-k} (p_a^c)^k$

pr. that k *backlogged* nodes transmit  $Q_r^c(k, n) = \binom{n}{k} (1-p_r)^{n-k} p_r^k$

$p_a^c$  : pr. at least one packet arrives at an unbacklogged node during two slots  
for Poisson arrival with rate  $\lambda$

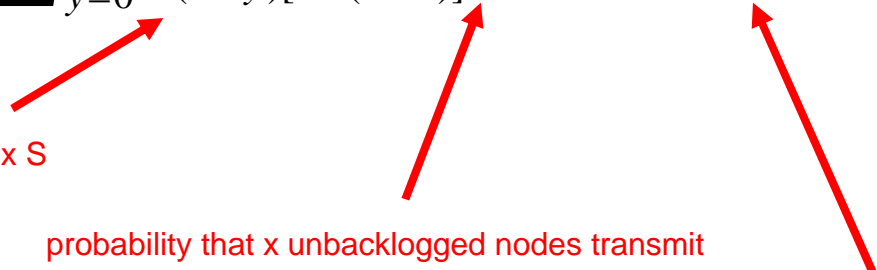
$p_r$  : retransmission pr. for a backlogged node during uplink

# Markovian Analysis

- Centrally Controlled Network -

(Finally) **transition probability**  $p_{nk}^c$  is given by:

$$p_{nk}^c = \begin{cases} \sum_{y=n-k}^n \sum_{x=0}^{M-n} S_{(x+y)[x+(n-k)]} Q_r^c(y, n) Q_a^c(x, n) & 0 \leq k < n \\ \sum_{x=k-n}^{M-n} \sum_{y=0}^n S_{(x+y)[x-(k-n)]} Q_a^c(x, n) Q_r^c(y, n) & n \leq k \leq M \end{cases}$$



reception matrix S

probability that x unbacklogged nodes transmit

probability that y backlogged nodes transmit

Defined Markov chain is *irreducible* and *aperiodic*, so we can obtain the **stationary distribution**  $\{\bar{q}_{j=0}^c\}$  of the network state

by solving the balance equation:  $\bar{q}^c = \bar{q}^c * P_c$

# Markovian Analysis

- Ad Hoc Network -

Network state can change during **one** time slot because of direct transmission.

The reception matrix  $S$  for the Centrally Controlled system does not completely characterize the multiple packet reception capability for the Ad Hoc system:

For example:

Node B cannot receive node A's packet because of half-duplex operation of transceiver.

Furthermore if node C successfully detects one packet, there is only half chance that the packet from B is detected.

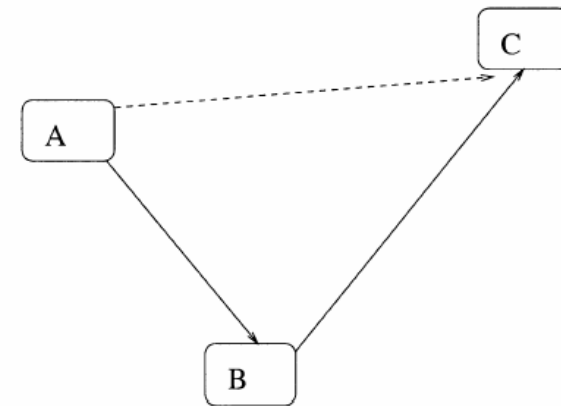


Fig. 2. Two issues in DS/SS slotted Aloha ad hoc networks.

# Markovian Analysis

- Ad Hoc Network -

We define the **reception matrix  $R$**  for the Ad hoc network:

$$R = \begin{pmatrix} r_{10} & r_{11} & 0 & \dots & 0 \\ r_{20} & r_{21} & r_{22} & \dots & 0 \\ \dots & & & & \dots \\ \dots & & & r_{(M-1)(M-1)} & 0 \\ r_{M1} & r_{M2} & r_{M3} & \dots & r_{MM} \end{pmatrix}$$

In general,  $R$  is a function of  $S$  and the network traffic pattern.

The conversion of  $S$  to  $R$  is given by a Theorem (see Paper Appendix A)

Making use of the theorem we get the matrix  $R$  and can now just substitute  $s_{ij}$  by  $r_{ij}$  in the formula for the transition probability of CC system

Transmission pr. of unbacklogged and backlogged nodes in Ad hoc system:

$$Q_a^a(k, n) = \binom{M-n}{k} (1-p_a^a)^{M-n-k} (p_a^a)^k \quad Q_r^a(k, n) = \binom{n}{k} (1-p_r)^{n-k} p_r^k$$



# Markovian Analysis

- Ad Hoc Network -

So we retrieve **transition probability**  $p_{nk}^a$  for the Ad hoc system:

$$p_{nk}^a = \begin{cases} \sum_{y=n-k}^n \sum_{x=0}^{M-n} r_{(x+y)[x+(n-k)]} Q_r^a(y, n) Q_a^a(x, n) & 0 \leq k < n \\ \sum_{x=k-n}^{M-n} \sum_{y=0}^n r_{(x+y)[x-(k-n)]} Q_a^a(x, n) Q_r^a(y, n) & n \leq k \leq M \end{cases}$$

reception matrix R  
for Ad hoc

probability that x unbacklogged nodes  
transmit

probability that y backlogged nodes transmit

Markov chain is irreducible and aperiodic.

Similar to CC system we can obtain the stationary distribution by solving the Markov chain balance equation:

$$\vec{q}^a = \vec{q}^a * P_a$$

# Performance Comparison

## Investigated criteria:

- Network throughput
- Average packet delay

Comparisons of these criteria are evaluated to determine the *effect of the Ad Hoc architecture* on the network performance

also:

Effects of **spreading gain N** and **error control coding t** which lead to the understanding of *efficiency of bandwidth utilization*

# Throughput Comparison

*Network throughput* defined as the average number of successfully received packets by their intended receiver in a time slot.

Throughput of Centrally Controlled network:

Given network state  $n$ , number of packets successfully received by their intended receivers in *two time slots*

$$N = \sum_{k=1}^M p_k^c \sum_{l=0}^k l s_{kl}$$

where

$$p_k^c = \sum_{x=0}^k Q_a^c(x, n) Q_r^c(k - x, n)$$

is the pr. that total  $k$  packets are transmitted in the uplink time slot

# Throughput Comparison

Because throughput  $\beta_c(n)$  and average throughput  $\bar{\beta}_c(n)$  are defined **per time slot** we get for the *centrally controlled* network:

$$\beta_c(n) = \frac{N}{2} \qquad \bar{\beta}_c = E(\beta(n)) = \sum_{n=0}^M \beta_c(n) q_n^c$$

Throughput of the Ad Hoc network:

$$\beta_a(n) = \sum_{k=1}^M p_k^a \sum_{l=0}^k l r_{kl} \qquad \bar{\beta}_a(n) = E(\beta_a(n)) = \sum_{n=0}^M \beta_a(n) q_n^a$$

where  $p_k^a$  is the pr. that total k packets are transmitted in one time slot

# Throughput Comparison

## Throughput bound for Ad Hoc network:

### *Intuition:*

If the receiver is *perfect*, i.e. all collided packages can be received successfully, throughput should reach the maximum.

Holds for the CC network but not for the Ad Hoc network.

*Reason:* half-duplex mode of transceivers impose limits on the throughput.

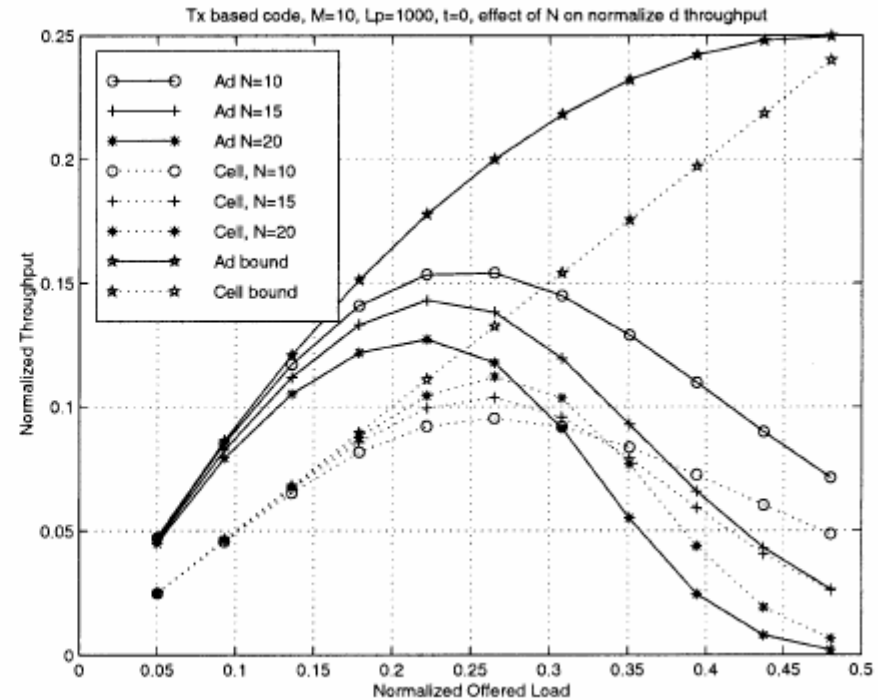
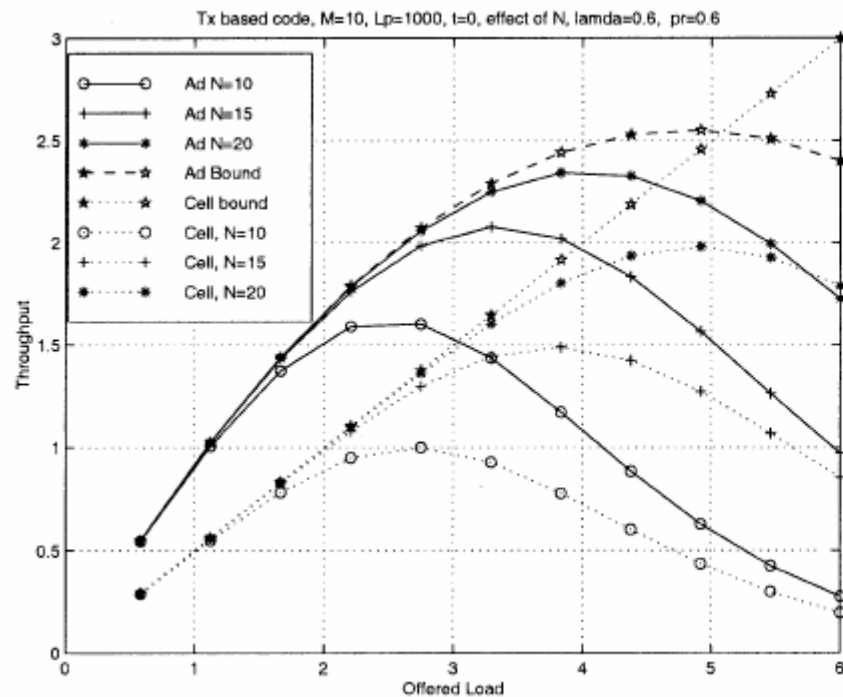
$$\beta_a(n) \leq \sum_{k=1}^M p_k \sum_{l=0}^k l q_l = \sum_{k=1}^M p_k \sum_{l=0}^k l \binom{k}{l} \left(\frac{M-L}{M-1}\right)^l \left(\frac{L-1}{M-1}\right)^{k-l}$$

## Normalization of Throughput:

$\beta_u$  : average number of information bits successfully received by their intended receivers per second per hertz

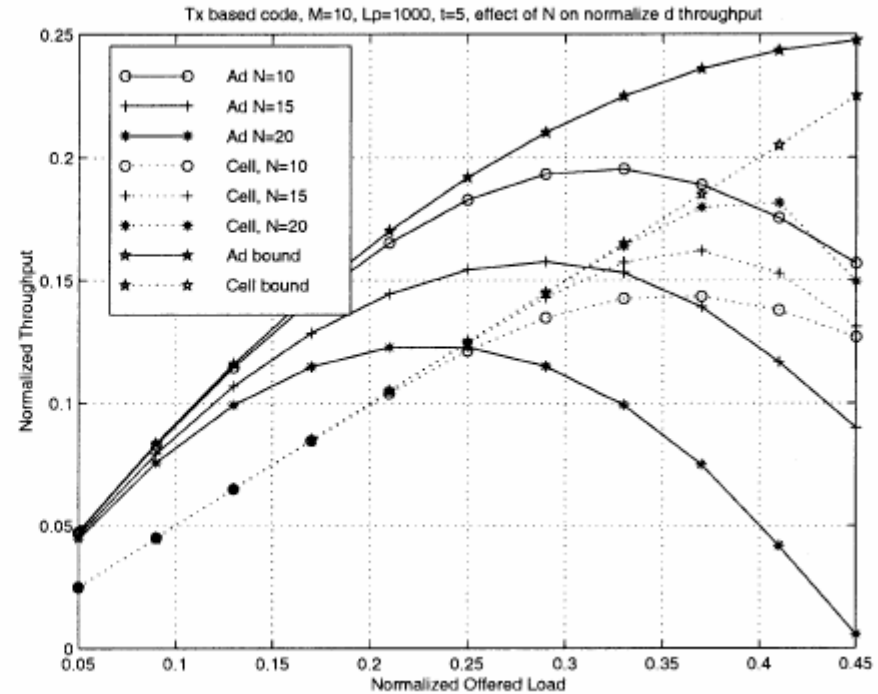
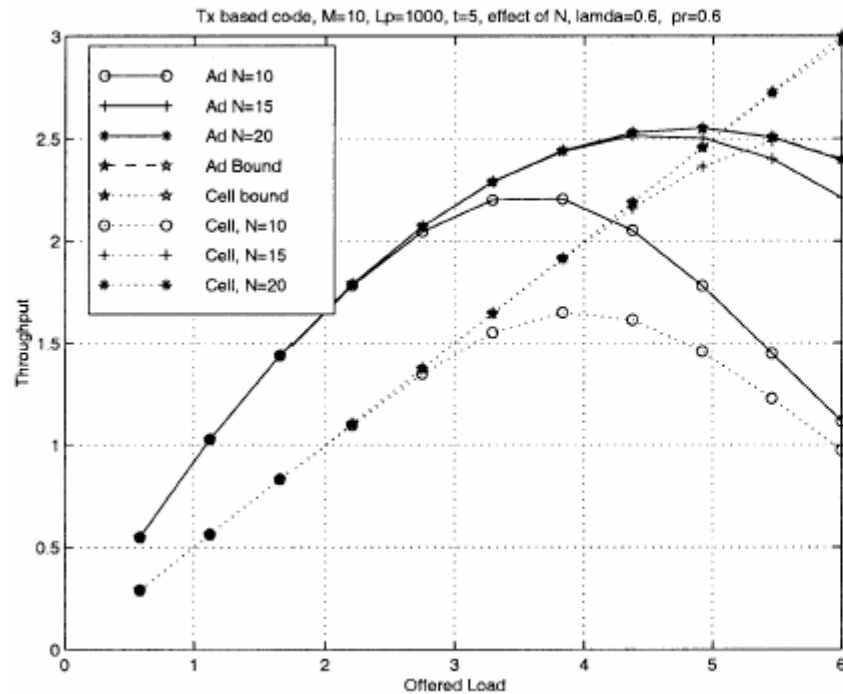
# Throughput Comparison

Throughput versus offered load;  $t=0$ ,  $\lambda=0.6$ ,  $p_r=0.6$



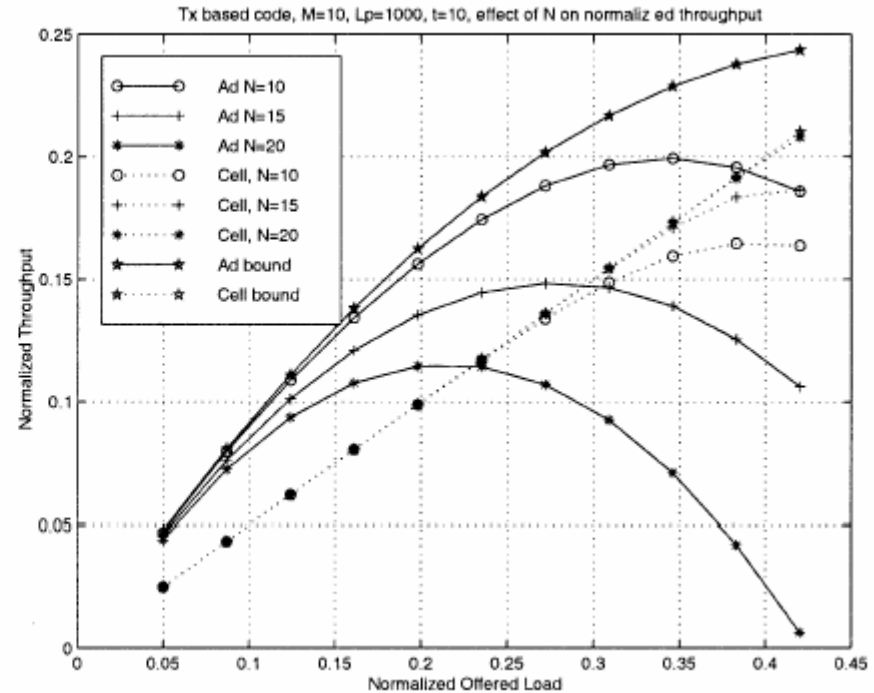
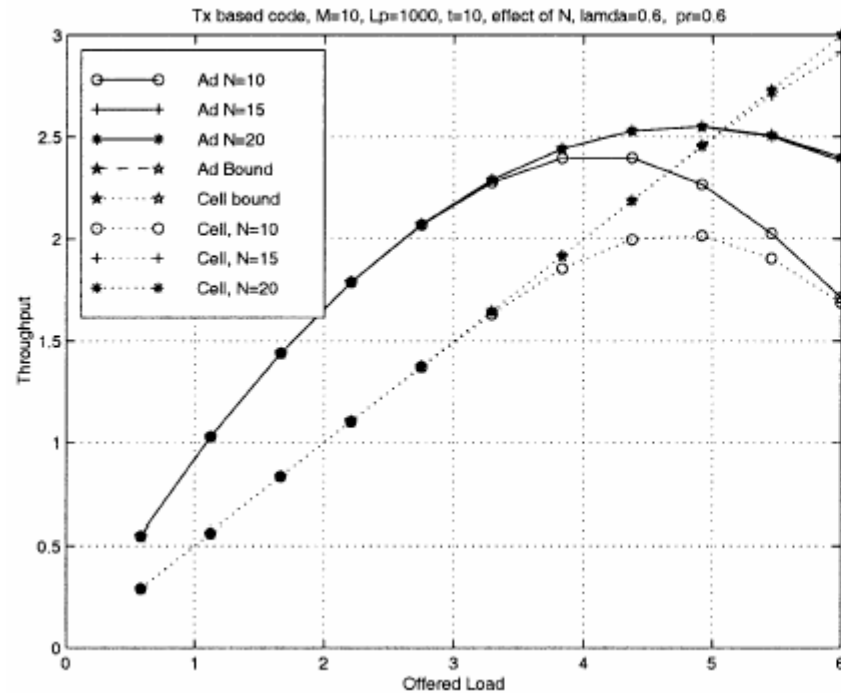
# Throughput Comparison

Throughput versus offered load;  $t=5$ ,  $\lambda=0.6$ ,  $p_r=0.6$



# Throughput Comparison

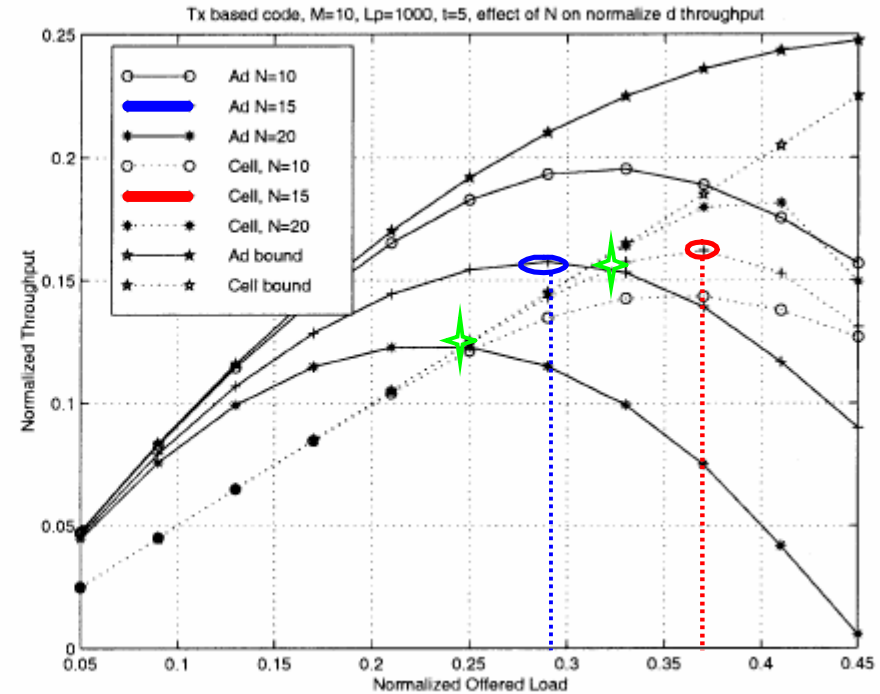
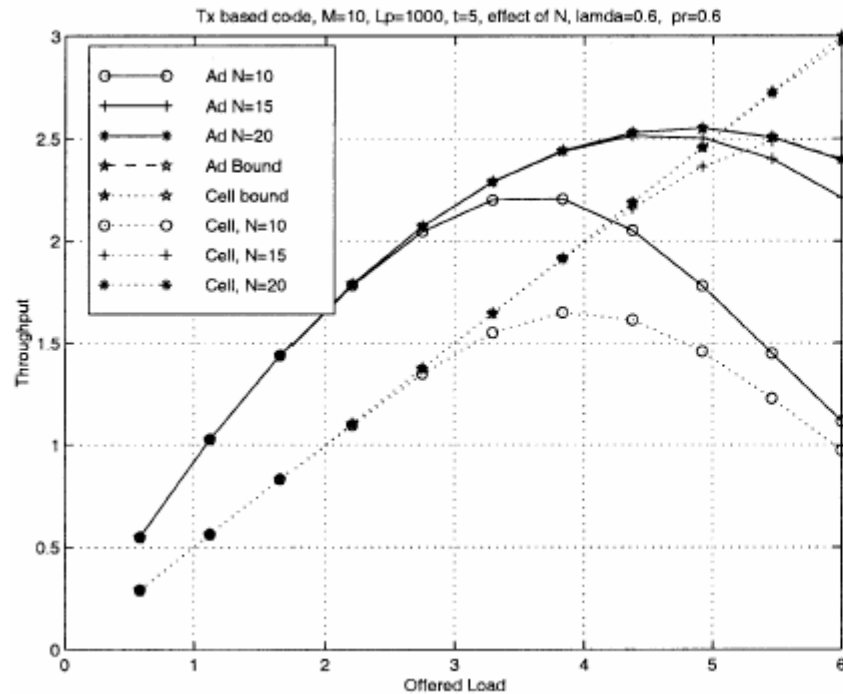
Throughput versus offered load;  $t=10$ ,  $\lambda=0.6$ ,  $p_r=0.6$





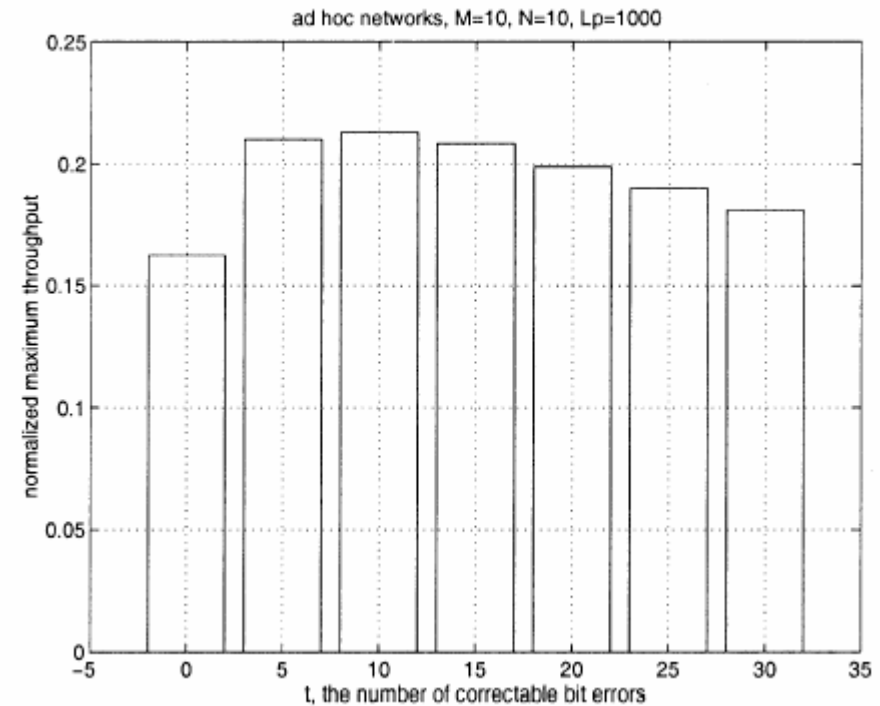
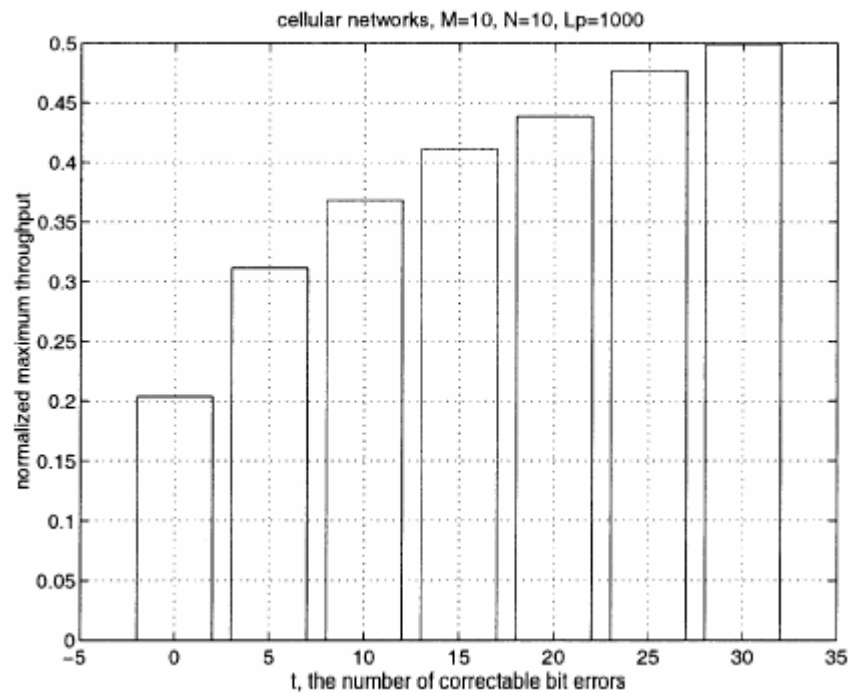
# Throughput Comparison

Throughput versus offered load;  $t=5$ ,  $\lambda=0.6$ ,  $p_r=0.6$



# Throughput Comparison

Number of correctable bit errors versus maximum normalized throughput



# Conclusion

- Performance of **Ad Hoc** network is negatively affected by two factors:
  - 1) Availability of node to receive packets
  - 2) Possibility that a node detected packets intended for other nodes
- Penalty caused by this issues is more significant when the network traffic is heavier
- These issues do not occur for the **Centrally Controlled** network *but* the process of relaying packets by the BS penalizes the network performance (especially at light traffic condition)

With constant spreading gain and error control coding:

*Light traffic:* Ad Hoc system had higher throughput and smaller packet delay

*Heavy traffic:* Centrally Controlled system outperforms the Ad Hoc system

# Conclusion

## Centrally Controlled network:

- With moderate powerful receivers, higher spreading gain can increase normalized throughput
- With poor receivers, higher spreading gain actually decreases normalized throughput under heavy traffic conditions
- Because of smaller bandwidth expansion introduced by the error control than the spreading gain, it is more efficient to use error control to improve centrally controlled networks

# Conclusion

## Ad Hoc network:

- Improvement of network throughput obtained from increased spreading gain cannot offset bandwidth expansion
- > increasing spreading gain monotonically decreases normalized throughput
- For given  $N$ , relationship between normalized throughput and correctable bit errors is not monotonic
- > optimum value for correctable bit errors can be selected to maximize the normalized throughput

# THE END

and thanks for listening...